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# ON WEAKLY $\omega$-CLOSED SETS IN TOPOLOGICAL SPACES

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**Abstract.** In this paper, the concepts of weakly $\omega$-closed sets and weakly $\omega$-open sets in a topological space, which are weaker forms of $\omega$-closed sets and $\omega$-open sets, are introduced. Many properties and relationships among those sets are investigated.

## 1. INTRODUCTION

Sundaram and Sheik John [20] have introduced the concept of $\omega$-closed sets and studied their most fundamental properties in topological spaces. In this paper, we will introduce a new class of generalized closed sets called weakly $\omega$-closed sets which contains the above mentioned class. Also, we will investigate the relationships among related generalized closed sets.

## 2. PRELIMINARIES

We recall the definitions, which are useful in the sequel.

**Definition 2.1.** *A subset A of a topological space X is called*
*(i) a semiopen set [10] if* $A \subseteq cl(int(A))$,
*(ii) a preopen set [15] if* $A \subseteq int(cl(A))$,
*(iii) an a-open set [17] if* $A \subseteq int(cl(int(A)))$,
*(iv) a semipreopen [1] if* $A \subseteq cl(int(cl(A)))$,
*(v) a regular open [19] if* $A = int(cl(A))$.
*The complements of the above mentioned sets are called a semiclosed set, a preclosed set, an $\alpha$-closed set, a semipreclosed set and regular closed set, respectively.*
*A semiclosure [7] (resp. $\alpha$-closure [16], semipreclosure [1]) of a subset A of X, denoted by scl(A) (resp. $\alpha cl(A)$, spcl(A)), is defined to be the intersection of all semiclosed (resp. $\alpha$-closed, semipreclosed) sets containing A. It is known that scl(A) (resp. $\alpha cl(A)$, spcl(A)) is a semiclosed (resp. $\alpha$-closed, semipreclosed) set. If $A \subseteq B \subseteq X$, then $cl_B(A)$ and $int_B(A)$ denotes the closure and the interior of A relative to B, respectively.*

**Definition 2.2.** *A subset A of a topological space X is called*
*(i) a generalized closed (briefly g-closed) set [9] if* $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open in X,*

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*(ii) a semigeneralized closed (briefly sg-closed) set [4] if scl(A)$\subseteq$U whenever A$\subseteq$U and U is semiopen in X,*

*(iii) a generalized semiclosed (briefly gs-closed) set [2] if scl(A)$\subseteq$U whenever A$\subseteq$U and U is open in X,*

*(iv) an $\alpha$-generalized closed (briefly $\alpha$g-closed) set [12] if $\alpha$cl(A)$\subseteq$U whenever A$\subseteq$U and U is open in X,*

*(v) a generalized $\alpha$-closed (briefly g$\alpha$-closed) set [13] if $\alpha$cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\alpha$-open in X,*

*(vi) a generalized semipreclosed (briefly gsp-closed) set [8] if spcl(A)$\subseteq$U whenever A$\subseteq$U and U is open in X,*

*(vii) an $\omega$-closed set [20] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semiopen in X.*

**Definition 2.3.** *Let X and Y be are topological space. A mapping $f : X \mapsto Y$ is called*

*(i) g-closed [14] if f(F) is g-closed set in Y for each closed set F in X,*

*(ii) regular closed [11] if f(F) is closed set in Y for each regular set F in X,*

*(iii) $\alpha$-continuous [16] if $f^{-1}(V)$ is $\alpha$-closed in X for each closed set V in Y,*

*(iv) gsp-continuous [8] if $f^{-1}(V)$ is gsp-closed in X for each closed set V in Y,*

*(v) completely continuous [3] (resp. R-map [5]) if $f^{-1}(V)$ is regular open in X for each open (resp. regular open) set V in Y,*

*(vi) perfectly continuous [18] if $f^{-1}(V)$ is both open and closed in X for each open set V in Y,*

*(vii) irresolute [6] f(V) is semiopen in Y for each semiopen set V in X.*

## 3. Weakly $\omega$-closed sets

We will introduce the notion of a weakly $\omega$-closed set in a topological space.

**Definition 3.1.** *A subset A of a topological space X is called a weakly $\omega$-closed set if cl(int(A))$\subseteq$G whenever A$\subseteq$G and G is semiopen in X.*

**Proposition 3.2.** *Every $\omega$-closed set is weakly $\omega$-closed.*

**Proposition 3.3.** *Every regular closed set is weakly $\omega$-closed.*

*Proof.* Let A be any regular closed set and let G be a semiopen set containing A. Since A is regular closed we have A=cl(int(A))$\subseteq$G. Thus, A is weakly $\omega$-closed. □

**Corrolary 3.4.** *Every closed set is weakly $\omega$-closed.*

**Remark 3.5.** The converse of the Propositions 3.2, 3.3 and the Corollary 3.4 need not be true, in general, as shown the following example.

**Example 3.6.** Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, X\}$. Then the set $\{b\}$ is weakly $\omega$-closed but none of regular closed, closed and $\omega$-closed in X.

**Proposition 3.7.** *Every weakly $\omega$-closed set is gsp-closed.*

*Proof.* Let A be any weakly $\omega$-closed set and G be an open set containing A. Then G is a semiopen set containing A, so cl(intA)$\subseteq$G. Since G is open we get int(cl(intA))$\subseteq$G which implies spcl(A)$\subseteq$G. Thus, A is gsp-closed in X. □

The converse of the Proposition 3.7 need not be true as shows the following example.

**Example 3.8.** We consider the topological space $(X, \tau)$ given in the Example 3.6. The subset $\{a, c\}$ is gsp-closed but it is not weakly $\omega$-closed in $(X, \tau)$.

**Proposition 3.9.** *If a subset A of a topological space X is both closed and $\alpha$g-closed, then it is weakly $\omega$-closed in X.*

*Proof.* Let A be any $\alpha$g-closed set in X and G is an open set containing A. Then A∪cl(int(cl(A)))=$\alpha$cl(A)⊆G. Since A is closed we have cl(int(A))⊆G, so A is weakly $\omega$-closed in X. □

**Proposition 3.10.** *If a subset A of a topological space X is both open and weakly $\omega$-closed, then it is closed.*

*Proof.* Since A is both open and weakly $\omega$-closed we have cl(A)=cl(int(A))⊆A. Hence A is closed in X. □

**Corrolary 3.11.** *If a subset A of a topological space X is both open and weakly $\omega$-closed, then it is both regular open and regular closed in X.*

**Theorem 3.12.** *A subset A is weakly $\omega$-closed in X if and only if cl(int(A))\A contains no non-empty semiclosed set in X.*

*Proof.* Let F be a semiclosed set such that F⊆cl(int(A))\A. Since $F^c$ is semiopen and A⊆ $F^c$, from the definition of weakly $\omega$-closed set it follows that cl(int(A))⊆ $F^c$. Thus F⊆ $(cl(int(A)))^c$. Therefore $F \subseteq cl(int(A)) \cap (cl(int(A))^c = \emptyset$.

Conversely, let A⊆G where G is a semiopen subset of X. If cl(int(A)) is not contained in G, then $cl(int(A)) \cap G^c$ is a non-empty semiclosed subset of cl(int(A))\A. It is a contradiction. □

**Theorem 3.13.** *Let X be a topological space and B⊆A⊆X. If B is weakly $\omega$-closed set relative to A and A is and weakly $\omega$-closed subset in X, then B is weakly $\omega$-closed set in X.*

*Proof.* Let B⊆G and G be semiopen in X. Then B⊆A∩G. Since B is weakly $\omega$-closed relative to A, we have clA(intA(B))⊆A∩G, so A∩cl(int(B))⊆A∩G. Thus A∩cl(int(B))⊆G. Then A∩cl(int(B))∪$(cl(int(B)))^c$ ⊆ G∪$(cl(int(B)))^c$. Since A is weakly $\omega$-closed in X, we have cl(int(A))⊆G∪$(cl(int(B))^c$. Since cl(int(B)) is not contained in $(cl(int(B)))^c$ we get cl(int(B))⊆G. Thus B is weakly $\omega$-closed set relative to X. □

**Corrolary 3.14.** *If A is weakly $\omega$-closed and F is closed in a topological space X, then A∩F is weakly $\omega$-closed in X.*

*Proof.* Since F is closed we have A∩F is closed in A. Therefore $cl_A(A \cap F) = A \cap F$ in A. Let $A \cap F$ ⊆G where G is semiopen in A. Then $cl_A(intA(A \cap F)) \subseteq G$, so A∩F is weakly $\omega$-closed in A. By the Theorem 3.13, A∩F is weakly $\omega$-closed in X. □

**Theorem 3.15.** *If $A$ is weakly $\omega$-closed and $A \subseteq B \subseteq cl(int(A))$, then $B$ is weakly $\omega$-closed.*

*Proof.* Since A⊆B we have $cl(int(B))\backslash B \subseteq cl(int(A))\backslash A$. By the Theorem 3.12 it follows that $cl(int(A))\backslash A$ contains no non-empty semiclosed set. It follows that $cl(int(B))\backslash B$ contains no non-empty semiclosed set, so B is weakly $\omega$-closed. □

**Theorem 3.16.** *Let $X$ be a topological space and $A \subseteq Y \subseteq X$. If $A$ is open and weakly $\omega$-closed in $X$, then $A$ is weakly $\omega$-closed relative to $Y$.*

*Proof.* Let A ⊆ Y∩G where G is semiopen in X. Since A is weakly $\omega$-closed in X, A ⊆ G implies that cl(int(A)) ⊆ G. It follows that Y∩(cl(int(A))⊆ Y∩G where Y∩cl(int(A)) is closure of interior of A in Y. Hence A is weakly $\omega$-closed relative to Y. □

**Theorem 3.17.** *Every nowhere dense subset $A$ of a topological space $X$ is weakly $\omega$-closed.*

*Proof.* Since int(A)⊆int(cl(A)) and A is nowhere dense, we have int(A)=∅. Therefore cl(int(A))=∅. Hence A is weakly $\omega$-closed in X. □

The converse of Theorem 3.17 need not be true as shows the following example.

**Example 3.18.** Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$. Then the set $\{a\}$ is weakly $\omega$-closed in $(X, \tau)$ but not nowhere dense in $(X, \tau)$.

**Remark 3.19.** Weakly $\omega$-closedness is independent of semi closedness, preclosedness, g-closedness and gs-closedness as shown by the following examples.

**Example 3.20.** Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then the set $\{a\}$ is semiclosed and gs-closed but not weakly $\omega$-closed.

**Example 3.21.** Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, b\}, X\}$. The subset $\{a, b\}$ is preclosed and g-closed but it is not weakly $\omega$-closed in $(X, \tau)$. The subset $\{c\}$ is weakly $\omega$-closed but it is not preclosed in $(X, \tau)$. Also the subset $\{a\}$ is weakly $\omega$-closed but it is not gs-closed and g-closed. Further, the set $\{b\}$ in the Example 3.18 is weakly $\omega$-closed in $(X, \tau)$ but it is not semiclosed.

**Definition 3.22.** *A subset $A$ of a topological space $X$ is called weakly $\omega$-open set if $A^c$ is weakly $\omega$-closed in $X$.*

**Proposition 3.23.** *Every open set is weakly $\omega$-open.*

*Proof.* Let A be any open set in a topological space X. Then $A^c$ is closed in X. From the Corollary 3.4 follows that $A^c$ is weakly $\omega$-closed in X. Hence A is weakly $\omega$-open in X. □

**Remark 3.24.** The converse of the Proposition 3.23 need not be true in general. We consider the topological space $(X, \tau)$ given in the Example 3.6. In this topological space the subset $\{a, c\}$ is weakly $\omega$-open but it is not open in X.

**Proposition 3.25.** *(i) Every $\omega$-open set in is weakly $\omega$-open.*

*(ii) Every regular open set is weakly $\omega$-open.*

*(iii) Every g-open set is weakly $\omega$-open.*

*(iv) Every weakly $\omega$-open set is gsp-open.*

It can be shows that the converse of (i), (ii), (iii) and (iv) need not be true.

**Theorem 3.26.** *A subset A of a topological space X is weakly $\omega$-open if $G \subseteq int(cl(A))$ whenever $G \subseteq A$ and G is semiclosed.*

*Proof.* Let A be any weakly $\omega$-open. Then $A^c$ is weakly $\omega$-closed. Let G be a semiclosed set contained in A. Then $G^c$ is a semiopen set in X containing $A^c$. Since $A^c$ is weakly $\omega$-closed we have cl(int($A^c$))$\subseteq G^c$. Therefore G$\subseteq$int(cl(A)).

Conversely, we suppose that G$\subseteq$int(cl(A)) whenever G$\subseteq$A and G is semiclosed. Then $G^c$ is a semiopen set containing $A^c$ and $G^c \supseteq int(cl(A^c))$. It follows that G$\subseteq$cl(int(A)), so A is weakly $\omega$-open. □

## 4. $\omega^*$-Continuous mappings

**Definition 4.1.** *Let X and Y are topological spaces. A mapping $f : X \mapsto Y$ is called $\omega^*$-continuous if $f^{-1}(U)$ is a weakly $\omega$-open set in X, for each open set U in Y.*

**Example 4.2.** Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. The mapping $f : (X, \tau) \mapsto (Y, \sigma)$ defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ is $\omega^*$-continuous, because every subset of X is weakly $\omega$-closed.

**Proposition 4.3.** *Every continuous mapping is $\omega^*$-continuous.*

*Proof.* It follows from the Proposition 3.23. □

The converse of the Proposition 4.3 need not be true as shows the following example.

**Example 4.4.** Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{b\}, Y\}$. Let $f : (X, \tau) \mapsto (Y, \sigma)$ be the identity mapping. Then f is $\omega^*$-continuous but it is not continuous.

**Proposition 4.5.** *Every $\omega$-continuous mapping is $\omega^*$-continuous.*

*Proof.* It follows from the Proposition 3.25 (i). □

The converse of the Proposition 4.5 need not be true as shows following example.

**Example 4.6.** Let $X, Y$ and $\tau$ as in the Example 4.4 and $\sigma = \{\emptyset, \{a\}, Y\}$. Define a function $f : (X, \tau) \mapsto (Y, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then $f$ is $\omega^*$-continuous but not $\omega$-continuous.

**Proposition 4.7.** *Every completely continuous mapping is $\omega^*$-continuous.*

*Proof.* It follows from the Proposition 3.25 (ii). □

**Remark 4.8.** The converse of the Proposition 4.7 need not be true in general. The mapping f in the Example 4.6 is $\omega^*$-continuous but it is not completely continuous.

**Proposition 4.9.** *Every $\omega^*$-continuous mapping is gsp-continuous.*

*Proof.* It follows from the Proposition 3.25 (iii). □

The converse of Proposition 4.9 need not be true as shows the following example.

**Example 4.10.** Let $X = Y = \{a,b,c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a,b\}, X\}$ and $\sigma = \{\emptyset, \{b,c\}, Y\}$. Let $f : (X,\tau) \mapsto (Y,\sigma)$ be the identity mapping. Hence $f$ is gsp-continuous but it is not $\omega^*$-continuous.

**Theorem 4.11.** *A mapping $f : X \mapsto Y$ is called $\omega^*$-continuous if and only if $f^{-1}(U)$ is a weakly $\omega$-closed set in X, for each closed set $U$ in $Y$.*

*Proof.* Let U be any closed set in Y. According to the assumption $f^{-1}(U^c) = X \backslash f^{-1}(U)$ is weakly $\omega$-open in X, so $f^{-1}(U)$ is weakly $\omega$-closed in X.

The converse can be prove in a similar manner. □

**Proposition 4.12.** *If $f : X \mapsto Y$ is perfectly continuous and $\omega^*$-continuous, then it is R-map.*

*Proof.* Let V be any open subset of Y. According to the assumption, $f^{-1}(V)$ is both open and closed in X. Since $f^{-1}(V)$ is closed it is weakly $\omega$-closed. Then $f^{-1}(V)$ is both open and weakly $\omega$-closed. Hence by Corollary 3.11 it is regular open in X, so f is R-map. □

**Definition 4.13.** *A topological space X is $\omega^*$-compact if every $\omega^*$-open cover of X has a finite subcover.*

**Theorem 4.14.** *Let $f : X \mapsto Y$ be a surjective $\omega^*$-continuous mapping. If X is $\omega^*$-compact, then Y is compact.*

*Proof.* Let $\{A_i : i \in I\}$ be an open cover of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a $\omega^*$-open cover of X. Since X is $\omega^*$-compact, it has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2), \ldots, f^{-1}(A_n)\}$. Since $f$ is surjective $\{A_1, A_2, \ldots, A_n\}$ is an open cover of Y and hence Y is compact. □

**Definition 4.15.** *A topological space X is $\omega^*$-connected if X cannot be written as the disjoint union of two non-empty weakly $\omega$-open sets.*

**Theorem 4.16.** *For a topological space X the following statements are equivalent:*

*(i) X is $\omega^*$-connected,*

*(ii) The empty set $\emptyset$ and X are only subsets which are both weakly $\omega$-open and weakly $\omega$-closed,*

*(iii) Each $\omega^*$-continuous mapping from X into a discrete space Y wihich has at least two points is a constant function.*

*Proof.* (i)⇒(ii) Let S⊂X be any proper subset, which is both weakly $\omega$-open and weakly $\omega$-closed. Its complement X\S is also weakly $\omega$-open and weakly $\omega$-closed. Then $X = S \cup (X \backslash S)$ is a disjoint union of two non-empty weakly $\omega$-open sets which is a contradiction with the fact that X is $\omega^*$-connected. Hence, $S = \emptyset$ or X.

(ii)⇒(i) Let $X = A \cup B$ where $A \cap B = \emptyset$, $A \neq \emptyset$ , $B \neq \emptyset$ and A, B are weakly $\omega$-open. Since $A = X \backslash B$, A is weakly $\omega$-closed. According to the assumption $A = \emptyset$ , which is a contradiction.

(ii)⇒(iii) Let $f : X \mapsto Y$ be a $\omega^*$-continuous mapping where Y is a discrete space with at least two points. Then $f^{-1}(\{y\})$ is weakly $\omega$-closed and weakly $\omega$-open for each $y \in Y$ and $X = \cup\{f^{-1}(\{y\})|y \in Y\}$. According to the assumption, $f^{-1}(\{y\}) = \emptyset$ or $f^{-1}(\{y\}) = X$. If $f^{-1}(\{y\}) = \emptyset$ for all $y \in Y$, $f$ will not be a mapping. Also there is no exist more than one $y \in Y$ such that $f^{-1}(\{y\}) = X$. Hence, there exists only one $y \in Y$ such that $f^{-1}(\{y\}) = X$ and $f^{-1}(\{y_1\}) = \emptyset$ where $y \neq y_1 \in Y$. This shows that $f$ is a constant mapping.

(iii)⇒(ii) Let $S \neq \emptyset$ be both weakly $\omega$-open and weakly $\omega$-closed in X. Let $f : X \mapsto Y$ be a $\omega^*$-continuous mapping defined by $f(S) = \{a\}$ and $f(X\backslash S) = \{b\}$ where $a \neq b$. Since $f$ is constant mapping we get $S = X$. □

**Theorem 4.17.** *Let $f : X \mapsto Y$ is $\omega^*$-continuous surjective mapping. If X is $\omega^*$-connected, then Y is connected.*

*Proof.* We suppose that Y is not connected. Then $Y = A \cup B$ where $A \cap B = \emptyset$, $A \neq \emptyset$, $B \neq \emptyset$ and A,B are open sets in Y. Since $f$ is $\omega^*$-continuous surjective mapping $X = f^{-1}(A) \cup f^{-1}(B)$ are disjoint union of two non-empty weakly $\omega$-open subsets. This is contradiction with the fact that X is $\omega^*$-connected. □

## 5. $\omega^*$-OPEN MAPPINGS AND $\omega^*$-CLOSED MAPPINGS

**Definition 5.1.** *Let X and Y are topological spaces. A mapping $f : X \mapsto Y$ is called $\omega^*$-open if $f(V)$ is a weakly $\omega$-open set in Y, for each open set V in X.*

**Definition 5.2.** *Let X and Y are topological spaces. A mapping $f : X \mapsto Y$ is called $\omega^*$-closed if $f(V)$ is a weakly $\omega$-closed in Y, for each closed set V in X.*

It is clear that an open mapping is $\omega^*$-open and a closed mapping is $\omega^*$-closed.

**Theorem 5.3.** *Let X and Y are topological spaces. A mapping $f : X \mapsto Y$ is $\omega^*$-closed if and only if for each subset B of Y and for each open set G containing $f^{-1}(B)$ there exists a weakly $\omega$-open set F of Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq G$.*

*Proof.* Let B be any subset of Y and let G be an open subset of X such that $f^{-1}(B) \subseteq G$. Then $F = Y\backslash f(X\backslash G)$ is weakly $\omega$-open set containing B and $f^{-1}(F) \subseteq G$.

Conversely, let U be any closed subset of X. Then $f^{-1}(Y\backslash f(U)) \subseteq X\backslash U$ and $X\backslash U$ is open. According to the assumption, there exists a weakly $\omega$-open set F of Y such that $Y\backslash f(U) \subseteq F$ and $f^{-1}(F) \subseteq X\backslash U$. Then $U \subseteq X\backslash \mathrm{f}^{-1}(F)$. From $Y\backslash F \subseteq f(U) \subseteq f(X\backslash f^{-1}(F)) \subseteq Y\backslash F$ follows that $f(U) = Y\backslash F$, so $f(U)$ is weakly $\omega$-closed in Y. Therefore $f$ is a $\omega^*$-closed mapping. □

**Theorem 5.4.** *If $f : X \mapsto Y$ is irresolute and $\omega^*$-closed mapping and A is a weakly $\omega$-closed set in X, then f(A) is weakly $\omega$-closed in Y.*

*Proof.* Let $f(A) \subseteq U$, where U is a semiopen set of Y. According to the assumption $f^{-1}(U)$ is semiopen in X, so cl(int(A))⊆ $f^{-1}(U)$. Since f is $\omega^*$-closed we can conclude that f(cl(int(A))) is weakly a $\omega$-closed set contained in the semiopen set U, which implies cl(int(f(cl(int(A))))⊆U. Therefore cl(int(f(A))⊆U. Hence f(A) is weakly $\omega$-closed in Y. □

**Corrolary 5.5.** *If $f : X \mapsto Y$ is irresolute and closed mapping and $A$ is a weakly $\omega$-closed set in $X$, then $f(A)$ is weakly $\omega$-closed in $Y$.*

**Theorem 5.6.** *If $f : X \mapsto Y$ is irresolute and $\omega^*$-closed mapping and $A$ is a weakly $\omega$-closed set in $X$, then $f_A : A \mapsto Y$ is $\omega^*$-closed mapping.*

*Proof.* Let F be any closed subset of A. Then F is weakly $\omega$-closed in X. From the Theorem 5.4 follows that $f_A(F) = f(F)$ is weakly $\omega$-closed in Y. Hence $f_A : A \mapsto Y$ is $\omega^*$-closed. □

**Remark 5.7.** The composition of two $\omega^*$-closed mappings need not be $\omega^*$-closed as we can see from the following example.

**Example 5.8.** Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$ and $\eta = \{\emptyset, \{a, b\}, Z\}$. We define $f : (X, \tau) \mapsto (Y, \sigma)$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$ and $g : (Y, \sigma) \mapsto (Z, \eta)$ be the identity map. Hence both $f$ and $g$ are $\omega^*$-closed mappings. For a closed set $U = \{b, c\}$, $(g \circ f)(U) = g(f(U)) = g(\{a, b\}) = \{a, b\}$ which is not weakly $\omega$-closed in Z. Hence the composition of two $\omega^*$-closed mappings need not be weakly $\omega$-closed.

**Theorem 5.9.** *Let X, Y and Z are topological spaces. If $f : X \mapsto Y$ be a closed mapping and $g : Y \mapsto Z$ be a $\omega^*$-closed map, then $g \circ f : X \mapsto Z$ is a $\omega^*$-closed mapping.*

**Definition 5.10.** *A mapping $f : X \mapsto Y$ is called a $\omega^*$-irresolute mapping if $f^{-1}(U)$ is a weakly $\omega$-open set in X, for each weakly $\omega$-open set U in X.*

**Example 5.11.** A mapping $f : X \mapsto Y$ in the Example 4.2 is $\omega^*$-irresolute.

**Remark 5.12.** The following examples show that irresoluteness and $\omega^*$-irresoluteness are independent.

**Example 5.13.** Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$, $\sigma = \{\emptyset, \{a\}, Y\}$ and $f : X \mapsto Y$ be the identity mapping. Then $f$ is $\omega^*$-irresolute but not irresolute.

**Example 5.14.** Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Let $f : (X, \tau) \mapsto (Y, \sigma)$ be the identity map. Then $f$ is irresolute but not $\omega^*$-irresolute.

**Theorem 5.15.** *The composition of two $\omega^*$-irresolute maps is also $\omega^*$-irresolute.*

**Theorem 5.16.** *Let $f : X \mapsto Y$ and $g : Y \mapsto Z$ are mapping such that $g \circ f : X \mapsto Z$ is $\omega^*$-closed mapping. Then the following statements hold:*
*(i) if $f$ is continuous and injective, then $g$ is $\omega^*$-closed.*
*(ii) if $g$ is $\omega^*$-irresolute and injective, then $f$ is $\omega^*$-closed.*

*Proof.* (i) Let $F$ be a closed set of $Y$. Since $f^{-1}(F)$ is closed in $X$ we can conclude that $(g \circ f)(f^{-1}(F))$ is weakly $\omega$-closed in $Z$. Hence $g(F)$ is weakly $\omega$-closed in $Z$. Thus $g$ is a $\omega^*$-closed mapping.

(ii) It can be prove in a similar manner as (i). □

**Theorem 5.17.** *If $f : X \mapsto Y$ is an $\omega^*$-irresolute mapping, then it is $\omega^*$-continuous.*

**Remark 5.18.** The converse of the above need not be true in general. The mapping $f : X \mapsto Y$ in the Example 4.6 is $\omega^*$-continuous but not $\omega^*$-irresolute.

**Theorem 5.19.** *If $f : X \mapsto Y$ is bijective semiopen and $\omega^*$-continuous mapping, then $f$ is $\omega^*$-irresolute.*

*Proof.* Let $F$ be any weakly $\omega$-closed set in $Y$ and $f^{-1}(F) \subseteq U$, where $U$ is a semiopen set in $X$. Then $F \subseteq f(U)$ and cl(int($F$))$\subseteq f(U)$. It follows that $f^{-1}$(cl(int($F$))$\subseteq U$. Since $f$ is $\omega^*$-continuous and cl(int($F$)) is closed in $Y$, $f^{-1}$(cl(int($F$))) is weakly $\omega$-closed in $X$. Since $f^{-1}$(cl(int($F$)))$\subseteq U$ and $f^{-1}$(cl(int($F$)) is weakly $\omega$-closed, we have cl(int($f^{-1}$(cl(int($F$))))$\subseteq U$, so cl(int($f^{-1}(F)$))$\subseteq U$. Therefore $f^{-1}(F)$ is weakly $\omega$-closed and hence $f$ is $\omega^*$-irresolute. □

**Theorem 5.20.** *If $f : X \mapsto Y$ is a surjective $\omega^*$-irresolute mapping and $X$ is $\omega^*$-compact, then $Y$ is $\omega^*$-compact.*

**Theorem 5.21.** *If $f : X \mapsto Y$ is surjective $\omega^*$-irresolute mapping and $X$ is $\omega^*$-connected, then $Y$ is $\omega^*$-connected.*


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