УШТЕ ЗА НЕКОИ ПРОБЛЕМИ СО СОПСТВЕНИ ВРЕДНОСТИ ОД IV РЕД

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Во овој труд ќе разгледаме повеќе проблеми со сопствени вредности од IV ред со одреден тип на контурните услови.

Поточно ќе ги разгледаме проблемите со сопствени вредности за равенката

$$y^{IV}+\lambda y = 0,\ \lambda=-k^4 \tag{1}$$

со контурните услови од обликот

$$y^{(r)}(a) = y^{(s)}(b) = 0 \tag{2}$$

$$my^{(r_1)}(a)+ny^{(s_1)}(b)=py^{(r_2)}(a)+qy^{(s_2)}(b)=0,$$

каде r и s земаат вредности 0 и 1 (т. е. само суштествени контурни услови); r_1, r_2; s_1, s_2 земаат вредности 0, 1, 2, 3 а $y^{(r)}(x) = \dfrac{d^r y}{dx^r}$.

Накусо, ќе ги разгледаме проблемите со сопствени вредности (1) и

(3) $(r;\ s;\ r_1\ s_1;\ r_2\ s_2)$. (Камкеовите ознаки)

Вакви проблеми се разгледувани во E. Kamke [1], L. Collatz [2] и др.

Овие проблеми со сопствени вредности (1), (3), можеме да ги добиеме непосредно од проблемот со сопствени вредности (1) и

$$\sum_{\nu=0}^{3}\left[\alpha_{\mu\nu}\, y^{(\nu)}(a) + \beta_{\mu\nu}\, y^{(\nu)}(b)\right] = 0, \qquad \mu = 1,\ 2,\ 3,\ 4 \tag{4}$$

за одредени посебни вредности на параметрите $\alpha_{\mu\nu}$ и $\beta_{\mu\nu}$, што сме го веќе детално обработиле во трудот [4].

Така за карактеристичната равенка за сопствените вредности на проблемот (1), (4) сме ја добиле равенката

$$
\begin{aligned}
& P_1(k)\operatorname{ch}K\cos K+[P_2(k)-\overline{P}_2(k)]\operatorname{ch}K\sin K\\
&+[P_3(k)-\overline{P}_3(k)]\operatorname{sh}K\cos K+[P_4(k)+\overline{P}_4(k)]\operatorname{sh}K\sin K\\
&+[P_5(k)+\overline{P}_5(k)]\cos K+[P_6(k)-\overline{P}_6(k)]\sin K\\
&+[P_7(k)+\overline{P}_7(k)]\operatorname{ch}K+[P_8(k)-\overline{P}_8(k)]\operatorname{sh}K+P_9(k)=0.
\end{aligned}
\tag{5}
$$

каде $K=k\,(b-a)$, додека $P_1(k), P_2(k), \ldots, P_9(k)$; $\overline{P}_1(k), \ldots, \overline{P}_8(k)$ се полиноми по k дадени со изразите:

$$
\begin{aligned}
P_1(k) = & -\det(\alpha_{12}\alpha_{23}\beta_{32}\beta_{43})k^8+[\det(\alpha_{20}\alpha_{21}\beta_{32}\beta_{43})\\
&+\det(\beta_{10}\beta_{21}\alpha_{32}\alpha_{43})-2\det(\alpha_{10}\alpha_{22}\beta_{31}\beta_{43})\\
&-2\det(\beta_{10}\beta_{22}\alpha_{31}\alpha_{43})+\det(\alpha_{10}\alpha_{23}\beta_{30}\beta_{43})\\
&+\det(\alpha_{10}\alpha_{23}\beta_{31}\beta_{42})+\det(\beta_{10}\beta_{23}\alpha_{31}\alpha_{42})\\
&+\det(\alpha_{11}\alpha_{22}\beta_{31}\beta_{42})]k^4-\det(\alpha_{10}\alpha_{21}\beta_{30}\beta_{41}),\\[6pt]
P_2(k) = & \det(\alpha_{11}\alpha_{23}\beta_{32}\beta_{43})k^7-[\det(\alpha_{10}\alpha_{23}\beta_{31}\beta_{43})\\
&+\det(\alpha_{11}\alpha_{22}\beta_{31}\beta_{43})+\det(\alpha_{10}\alpha_{22}\beta_{32}\beta_{43})]k^5\\
&+[\det(\alpha_{10}\alpha_{21}\beta_{31}\beta_{43})-\det(\alpha_{10}\alpha_{22}\beta_{31}\beta_{42})\\
&-\det(\alpha_{10}\alpha_{22}\beta_{30}\beta_{43})]k^3+\det(\alpha_{10}\alpha_{21}\beta_{30}\beta_{42})k,\\[6pt]
P_3(k) = & \det(\alpha_{11}\alpha_{23}\beta_{32}\beta_{43})k^7+[\det(\alpha_{10}\alpha_{22}\beta_{32}\beta_{43})\\
&+\det(\alpha_{10}\alpha_{23}\beta_{31}\beta_{43})+\det(\alpha_{11}\alpha_{22}\beta_{31}\beta_{43})]k^5\\
&+[\det(\alpha_{10}\alpha_{21}\beta_{31}\beta_{43})-\det(\alpha_{10}\alpha_{22}\beta_{31}\beta_{43})\\
&-\det(\alpha_{10}\alpha_{22}\beta_{30}\beta_{43})]k^3-\det(\alpha_{10}\alpha_{21}\beta_{30}\beta_{42})k,\\[6pt]
P_4(k) = & -[\det(\alpha_{10}\alpha_{23}\beta_{32}\beta_{43})+\det(\alpha_{11}\alpha_{22}\beta_{32}\beta_{43})\\
&-\det(\alpha_{11}\alpha_{23}\beta_{31}\beta_{43})]k^6+[\det(\alpha_{10}\alpha_{21}\beta_{30}\beta_{43})\\
&+\det(\alpha_{10}\alpha_{21}\beta_{31}\beta_{42})-\det(\alpha_{10}\alpha_{22}\beta_{30}\beta_{42})]k^2,\\[6pt]
P_5(k) = & [\det(\alpha_{11}\alpha_{22}\beta_{33}\beta_{42})-\det(\alpha_{10}\alpha_{22}\beta_{33}\beta_{43})]k^6\\
&-[\det(\alpha_{11}\alpha_{22}\beta_{33}\beta_{40})+\det(\alpha_{10}\alpha_{21}\alpha_{33}\beta_{43})\\
&-\det(\alpha_{10}\alpha_{21}\alpha_{32}\beta_{43})-\det(\alpha_{10}\alpha_{22}\alpha_{33}\beta_{41})]k^4\\
&+[\det(\alpha_{10}\alpha_{21}\alpha_{33}\beta_{40})-\det(\alpha_{10}\alpha_{21}\alpha_{32}\beta_{41})]k^2,
\end{aligned}
\tag{6}
$$

$$P_6(k) = -\det(\alpha_{11}\,\alpha_{22}\,\alpha_{33}\,\beta_{43})\,k^7 + [-\det(\alpha_{10}\,\alpha_{22}\,\alpha_{33}\,\beta_{42})$$
$$+ \det(\alpha_{10}\,\alpha_{21}\,\alpha_{33}\,\beta_{43}) + \det(\alpha_{11}\,\alpha_{22}\,\alpha_{33}\,\beta_{41})]\,k^5$$
$$+ [\det(\alpha_{10}\,\alpha_{21}\,\alpha_{32}\,\alpha_{42}) + \det(\alpha_{10}\,\alpha_{22}\,\alpha_{33}\,\beta_{40})$$
$$- \det(\alpha_{10}\,\alpha_{21}\,\alpha_{33}\,\beta_{41})]\,k^3 - \det(\alpha_{10}\,\alpha_{21}\,\alpha_{32}\,\beta_{40})\,k,$$

$$P_7(k) = [\det(\alpha_{10}\,\alpha_{22}\,\alpha_{33}\,\beta_{43}) - \det(\alpha_{11}\,\alpha_{22}\,\alpha_{33}\,\beta_{42})]\,k^6$$
$$+ [\det(\alpha_{10}\,\alpha_{21}\,\alpha_{32}\,\beta_{43}) + \det(\alpha_{10}\,\alpha_{22}\,\alpha_{33}\,\beta_{41})$$
$$- \det(\alpha_{11}\,\alpha_{22}\,\alpha_{33}\,\beta_{40}) - \det(\alpha_{10}\,\alpha_{21}\,\alpha_{33}\,\beta_{42})]\,k^4$$
$$+ [\det(\alpha_{10}\,\alpha_{21}\,\alpha_{32}\,\beta_{41}) - \det(\alpha_{10}\,\alpha_{21}\,\alpha_{33}\,\beta_{40})]\,k^2,$$

$$P_8(k) = -\det(\alpha_{11}\,\alpha_{22}\,\alpha_{33}\,\beta_{43})\,k^7 + [\det(\alpha_{10}\,\alpha_{22}\,\alpha_{33}\,\beta_{42})$$
$$- \det(\alpha_{10}\,\alpha_{22}\,\alpha_{33}\,\beta_{43}) - \det(\alpha_{11}\,\alpha_{22}\,\alpha_{33}\,\beta_{41})]\,k^5$$
$$+ [\det(\alpha_{10}\,\alpha_{21}\,\alpha_{32}\,\beta_{42}) + \det(\alpha_{10}\,\alpha_{22}\,\alpha_{33}\,\beta_{40})$$
$$- \det(\alpha_{10}\,\alpha_{21}\,\alpha_{33}\,\beta_{41})\,k^3 + \det(\alpha_{10}\,\alpha_{21}\,\alpha_{32}\,\beta_{40})\,k,$$

$$P_9(k) = \det(\alpha_{12}\,\alpha_{23}\,\beta_{32}\,\beta_{43})\,k^8 + [\det(\alpha_{10}\,\alpha_{21}\,\beta_{32}\,\beta_{43})$$
$$+ \det(\beta_{10}\,\beta_{21}\,\alpha_{32}\,\alpha_{43}) - \det(\alpha_{10}\,\alpha_{23}\,\beta_{30}\,\beta_{43})$$
$$+ \det(\alpha_{10}\,\alpha_{23}\,\beta_{31}\,\beta_{42}) + \det(\beta_{10}\,\beta_{23}\,\alpha_{31}\,\alpha_{42})$$
$$- \det(\alpha_{11}\,\alpha_{22}\,\beta_{31}\,\beta_{42}) + 2\det(\alpha_{10}\,\alpha_{21}\,\alpha_{32}\,\alpha_{43})$$
$$+ 2\det(\beta_{10}\,\beta_{21}\,\beta_{32}\,\beta_{43})]\,k^4 + \det(\alpha_{10}\,\alpha_{21}\,\beta_{30}\,\beta_{41}),$$

Ако во $P_2(k), \ldots, P_8(k)$ меѓусебно се сменат α и β ги добиваме полиномите $\overline{P}_2(k), \ldots, \overline{P}_8(k)$.

Поради поголема концизност на резултатите во понатамошниот текст ќе ги употребиме кратениците на Камке

$$\alpha = \cos K \operatorname{ch} K, \quad \gamma = \cos K \operatorname{sh} K$$
$$\beta = \sin K \operatorname{sh} K, \quad \delta = \sin K \operatorname{ch} K$$

како и

$$u_1(x, k) = \operatorname{ch} k\,(x - a) + \cos k\,(x - a)$$
$$u_2(x, k) = \operatorname{ch} k\,(x - a) - \cos k\,(x - a)$$
$$u_3(x, k) = \operatorname{sh} k\,(x - a) + \sin k\,(x - a)$$
$$u_4(x, k) = \operatorname{sh} k\,(x - a) - \sin k\,(x - a).$$

Диф. рав.	Контурните услови и посебните вредности на $\alpha_{\mu\nu}$ и $\beta_{\mu\nu}$	Трансцендентната равенка
$\lambda^{IV} = \lambda y$ $\lambda = k^4$	(0; 0; 11; 22) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{42} = p$ $\beta_{20} = 1,\ \beta_{31} = n,\ \beta_{32} = q$	$(mq + np)(\gamma - \delta) + (mp+nq)u_4(b,k) = 0$
	(0; 0; 11; 23) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{32} = p$ $\beta_{20} = 1,\ \beta_{31} = n,\ \beta_{43} = q$	$np(\gamma - \delta) - mqk\beta$ $+ nqu_2(b, k) - mpu_3(b, k) = 0$
	(0; 0; 11; 32) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{43} = p$ $\beta_{20} = 1,\ \beta_{31} = n,\ \beta_{42} = q$	$mq(\gamma - \delta) + npk\beta$ $- mpku_2(b, k) + nqu_3(b, k) = 0$
	(0; 0; 11; 33) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{43} = p$ $\beta_{20} = 1,\ \beta_{31} = n,\ \beta_{43} = q$	$(mq + np)\beta + (mp - nq)u_2(b, k) = 0$
	(0; 0; 12; 21) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{42} = p$ $\beta_{20} = 1,\ \beta_{32} = n,\ \beta_{41} = q$	$mq(\alpha - 1) - 2npk^2\beta$ $+ (mp - nq)ku_4(b, k) = 0$
	(0; 0; 12; 23) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{42} = p$ $\beta_{20} = 1,\ \beta_{32} = n,\ \beta_{43} = q$	$(2np + mq)\beta$ $- mpu_4(b, k) - nqk^2 u_3(b, k) = 0$

Диф. рав.	Контурните услови и посебните вредности на $\alpha_{\mu\nu}$ и $\beta_{\mu\nu}$	Трансцендентната равенка
$y^{IV}=\lambda y$	(0; 0; 12; 31)	$mq(\alpha-1)+npk^3(\gamma+\delta)$
$\lambda=k^4$	$\beta_{20}=1,\ \beta_{32}=m,\ \beta_{41}=p$	$-nqku_4(b,k)-mpk^2u_2(b,k)=0$
	$\beta_{20}=1,\ \beta_{32}=n,\ \beta_{41}=q$	
	(0; 0; 12; 33)	
	$\alpha_{10}=1,\ \alpha_{31}=m,\ \alpha_{43}=p$	$npk(\gamma+\delta)-mq\beta$
	$\beta_{20}=1,\ \beta_{32}=n,\ \beta_{43}=q$	$-mpu_2(b,k)+nqku_3(b,k)=0$
	(0; 0; 13; 21)	
	$\alpha_{10}=1,\ \alpha_{31}=m,\ \alpha_{42}=p$	$mq(\alpha-1)+npk_3(\gamma+\delta)$
	$\beta_{20}=1,\ \beta_{33}=n,\ \beta_{41}=q$	$-nqk^2u_2(b,k)+mpku_4(b,k)=0$
	(0; 0; 13; 22)	
	$\alpha_{10}=1,\ \alpha_{31}=m,\ \alpha_{42}=p$	$(npk^2+mq)\delta+(npk^2-mq)\gamma$
	$\beta_{20}=1,\ \beta_{33}=n,\ \beta_{42}=q$	$-mpu_4(b,k)+nqk^2u_3(b,k)=0$
	(0; 0; 13; 31)	
	$\alpha_{10}=1,\ \alpha_{31}=m,\ \alpha_{43}=p$	$(npk^4-mq)(\alpha-1)$
	$\beta_{20}=1,\ \beta_{33}=n,\ \beta_{41}=q$	$+(mp+nq)k^2u_1(b,k)=0$
	(0; 0; 13; 32)	
	$\alpha_{10}=1,\ \alpha_{31}=m,\ \alpha_{43}=p$	$npk^3(\alpha-1)+mq(\gamma-\delta)$
	$\beta_{20}=1,\ \beta_{33}=n,\ \beta_{42}=q$	$-mpku_2(b,k)-nqk^2u_3(b,k)=0$

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$y^{IV} = \lambda y$	(0; 0; 22; 31)	
$\lambda = k^4$	$\alpha_{10} = 1,\ \alpha_{32} = m,\ \alpha_{43} = p$	$(npk^2 + mq)\,\delta + (npk^2 - mq)\gamma$
	$\beta_{20} = 1,\ \beta_{32} = n,\ \beta_{41} = q$	$+ mpk^2 u_3(b, k) - nqu_4(b, k) = 0$
	(0; 0; 22; 33)	
	$\alpha_{10} = 1,\ \alpha_{32} = m,\ \alpha_{43} = p$	$(mq + np)(\gamma + \delta) + (mp + nq) u_3(b,k) = 0$
	$\beta_{20} = 1,\ \beta_{32} = n,\ \gamma_{43} = q$	
	(0; 0; 23; 31)	
	$\alpha_{10} = 1,\ \alpha_{32} = m,\ \alpha_{43} = p$	$npk^3(\alpha - 1) + mq(\delta - \gamma)$
	$\beta_{20} = 1,\ \beta_{33} = n,\ \beta_{41} = q$	$-nqku_2(b, k) + mpk^2 u_3(b, k) = 0$
	(0; 0; 23; 32)	
	$\alpha_{10} = 1,\ \alpha_{32} = m,\ \alpha_{43} = p$	$npk(\alpha - 1) - 2mq\beta$
	$\beta_{20} = 1,\ \beta_{33} = n,\ \beta_{42} = q$	$+ (mp - nq) ku_3(b, k) = 0$
	(0; 1; 10; 22)	
	$\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{42} = p$	$np(\gamma - \delta) + mqkp - mpku_2(b, k)$
	$\beta_{21} = 1,\ \beta_{30} = n,\ \beta_{42} = q$	$+ nqu_4(b, k) = 0$
	(0; 1; 10; 23)	
	$\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{42} = p$	$(mqk^2 - np)\,\delta + (mqk^2 + np)\gamma$
	$\beta_{21} = 1,\ \beta_{30} = n,\ \beta_{43} = q$	$-(mp - nq)\,u_2(b, k) = 0$

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$y^{IV} = \lambda y$ $\lambda = k^4$	(0; 1; 10; 32) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{43} = p$ $\beta_{21} = 1,\ \beta_{30} = n,\ \beta_{42} = q$	$(mq + np)\, k\beta$ $+ mpk^2 u_3 (b, k) + nqu_4 (b, k) = 0$
	(0; 1; 10; 33) $\alpha_{10} = 1,\ \alpha_{32} = m,\ \alpha_{43} = p$ $\beta_{21} = 1,\ \beta_{30} = n,\ \beta_{43} = q$	$mqk (\gamma + \delta) + np\beta$ $+ nqu_2 (b, k) + mpku_3 (b, k) = 0$
	(0; 1; 12; 20) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{42} = p$ $\beta_{21} = 1,\ \beta_{32} = n,\ \beta_{40} = q$	$mq (\alpha - 1) + mpk^3 (\gamma + \delta)$ $- mpk^2 u_2 (b, k) - nqk_4 (b, k) = 0$
	(0; 1; 12; 23) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{42} = p$ $\beta_{21} = 1,\ \beta_{32} = n,\ \beta_{43} = q$	$(mq + np)\, k (\gamma + \delta)$ $- mpu_2 (b, k) - nqk^2 u_1 (b, k) = 0$
	(0; 1; 12; 30) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{43} = p$ $\beta_{21} = 1,\ \beta_{32} = n,\ \beta_{40} = q$	$(npk_4 - mq)\, \alpha + npk^4 + mq$ $- mpk^3 u_3 (b, k) + nqku_4 (b, k) = 0$
	(0; 1; 12; 30) $\alpha_{10} = 1,\ \alpha_{31} = m,\ \alpha_{43} = p$ $\beta_{21} = 1,\ \beta_{32} = n,\ \beta_{43} = q$	$npk (\alpha + 1) - mq (\gamma + \delta)$ $+ nqku_1 (b, k) - mpu_3 (b, k) = 0$

Диф. рав.	Контурните услови и посебните вредности на $\alpha_{\mu\nu}$ и $\beta_{\mu\nu}$	Трансцендентната равенка
$y^{IV}=\lambda y$ $\lambda=k^4$	(0; 1; 13; 20) $\alpha_{10}=1, \alpha_{31}=m, \alpha_{42}=p$ $\beta_{21}=1, \beta_{33}=n, \beta_{40}=q$	$(2\,npk^4+mq)\,\alpha-mq$ $-k^2\,(mp+nqu)\,u_2\,(b, k)=0$
	(0; 1; 13; 22) $\alpha_{10}=1, \alpha_{31}=m, \alpha_{42}=p$ $\beta_{21}=1, \beta_{33}=n, \beta_{42}=q$	$2\,npk^2\,\alpha-mq\beta$ $+nqk^2\,u_1\,(b, k)+mpu_2\,(b, k)=0$
	(0; 1; 13; 30) $\alpha_{10}=1, \alpha_{31}=m, \alpha_{43}=p$ $\beta_{21}=1, \beta_{33}=n, \beta_{40}=q$	$mq\,(1-\alpha)+npk^5\,(\gamma-\delta)$ $+npk^2\,u_2\,(b, k)-mpk^3\,u_3\,(b, k)=0$
	(0; 1; 13; 32) $\alpha_{10}=1, \alpha_{31}=m, \alpha_{43}=p$ $\beta_{21}=1, \beta_{33}=n, \beta_{42}=q$	$npk^3\,(\delta-\gamma)+mq\beta$ $+npk^2\,u_1\,(b, k)+mpku_3\,(b, k)=0$
	(0; 1; 20; 32) $\alpha_{10}=1, \alpha_{32}=m, \alpha_{43}=p$ $\beta_{21}=1, \beta_{30}=n, \beta_{42}=q$	$mqk^2\,(\gamma+\delta)-npk\beta$ $+mpk^3\,u_2\,(b, k)-nqu_4\,(b, k)=0$
	(0; 1; 20; 33) $\alpha_{10}=1, \alpha_{32}=m, \alpha_{43}=p,$ $\beta_{21}=1, \beta_{30}=n, \beta_{43}=q$	$2\,mqk^2\,\alpha-np\beta$ $+mpk^2\,u_1\,(b, k)-nqu_2\,(b, k)=0$

Диф. рав.	Контурните услови и посебните вредности на $\alpha_{\mu\nu}$ и $\beta_{\mu\nu}$	Трансцендентната равенка
$y^{IV}=\lambda y$	(0; 1; 22; 30)	
$\lambda=k^4$	$\alpha_{10}=1,\ \alpha_{32}=m,\ \alpha_{43}=p$	$npk^3(\alpha+1)+mq(\gamma-\delta)$
	$\beta_{21}=1,\ \beta_{32}=n,\ \beta_{40}=q$	$+mpk^3u_1(b,k)+nqu_4(b,k)=0$
	(0; 1; 22; 33)	
	$\alpha_{10}=1,\ \alpha_{32}=m,\ \alpha_{43}=p$	$(2mq+np)\alpha+np$
	$\beta_{21}=1,\ \beta_{32}=n,\ \beta_{43}=q$	$+(mp+nq)u_1(b,k)=0$
	(0; 1; 23; 30)	
	$\alpha_{10}=1,\ \alpha_{32}=m,\ \alpha_{43}=p$	$(npk^4+mq)(\gamma-\delta)$
	$\beta_{21}=1,\ \beta_{33}=n,\ \beta_{40}=q$	$+mpk^3u_1(b,k)-nqku_2(b,k)=0$
	(0; 1; 23; 32)	
	$\alpha_{10}=1,\ \alpha_{32}=m,\ \alpha_{43}=p$	$(mq-npk^2)\delta+(nq+npk^2)\gamma$
	$\beta_{21}=1,\ \beta_{33}=n,\ \beta_{42}=q$	$+(mp-nq)ku_1(b,k)=0$
	(1; 1; 00; 22)	
	$\alpha_{10}=1,\ \alpha_{30}=m,\ \alpha_{42}=p$	$(mp-np)\beta+(nq-mp)u_2(b,k)=0$
	$\beta_{21}=1,\ \beta_{30}=n,\ \beta_{42}=q$	
	(1; 1; 00; 23)	
	$\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{42}=p$	$mqk(\gamma+\delta)-np\beta$
	$\beta_{21}=1,\ \beta_{30}=n,\ \beta_{43}=q$	$-mpu_2(b,k)+nqku_3(b,k)=0$

Диф. рав.	Контурните услови и посебните вредности на $\alpha_{\mu\nu}$ и $\beta_{\mu\nu}$	Трансцендентната равенка
$y^{IV}=\lambda y$ $\lambda=k^4$	(1; 1; 00; 32) $\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{43}=p$ $\beta_{21}=1,\ \beta_{30}=n,\ \beta_{42}=q$	$npk(\gamma+\delta)+mq\beta$ $+nqu_2(b,k)+mpku_3(b,k)=0$
	(1; 1; 00; 33) $\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{43}=p$ $\beta_{21}=1,\ \beta_{30}=n,\ \beta_{43}=q$	$(mq+np)(\gamma+\delta)$ $+(mp+nq)u_3(b,k)=0$
	(1; 1; 02, 20) $\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{42}=p$ $\beta_{21}=1,\ \beta_{32}=n,\ \beta_{40}=q$	$(npk^4+mq)(\alpha-1)$ $-(mp+nq)k^2u_2(b,k)=0$
	(1; 1; 02; 23) $\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{42}=p$ $\beta_{21}=1,\ \beta_{32}=n,\ \beta_{43}=q$	$npk^3(\alpha-1)+mqk(\gamma+\delta)$ $-mpu_2(b,k)-nqk^3u_4(b,k)=0$
	(1; 1; 02; 30) $\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{43}=p$ $\beta_{21}=1,\ \beta_{32}=n,\ \beta_{40}=q$	$mq(\alpha-1)+npk^5(\delta-\gamma)$ $-nqk^2u_2(b,k)+mpk^2u_3(b,k)=0$
	(1; 1; 02; 33) $\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{43}=p$ $\beta_{21}=1,\ \beta_{32}=n,\ \beta_{43}=q$	$(mq+npk^2)\delta+(mq-npk^2)\gamma$ $+mpu_3(b,k)-nqk^2u^4(b,k)=0$

Диф. рав.	Контурните услови и посебните вредности на и $\alpha_{\mu\nu}$ $\beta_{\mu\nu}$	Трансцендентната равенка
$y^{IV}=\lambda y$	(1; 1; 03; 20)	
$\lambda=k^4$	$\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{42}=p$	$mq(\alpha-1)+npk^5(\gamma-\delta)$
	$\beta_{21}=1,\ \beta_{33}=n,\ \beta_{40}=q$	$-mpk^2u_2(b,k)-nqk^3u_3(b,k)=0$
	(1; 1; 03; 22)	
	$\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{42}=p$	$npk^3(\gamma-\delta)+mq\beta$
	$\beta_{21}=1,\ \beta_{33}=n,\ \beta_{42}=q$	$-mpu_2(b,k)+nqk^3u_4(b,k)=0$
	(1; 1; 03; 30)	
	$\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{43}=p$	$mq(\alpha-1)+2\,npk^6\beta$
	$\beta_{21}=1,\ \beta_{33}=n,\ \beta_{40}=q$	$+(mp-nq)k^3u_3(b,k)=0$
	(1; 1; 03; 32)	
	$\alpha_{11}=1,\ \alpha_{30}=m,\ \alpha_{43}=p$	$(2npk^4+mq)\beta+mpku_3(b,k)$
	$\beta_{21}=1,\ \beta_{33}=n,\ \beta_{42}=q$	$+nqk^3u_4(b,k)=0$
	(1; 1; 22; 33)	
	$\alpha_{11}=1,\ \alpha_{32}=m,\ \alpha_{43}=p$	$(mq+np)(\gamma-\delta)$
	$\beta_{21}=1,\ \alpha_{32}=n,\ \beta_{43}=q$	$+(mp+nq)u_4(b,k)=0$
	(1; 1; 23; 32)	
	$\alpha_{11}=1,\ \alpha_{32}=m,\ \alpha_{43}=p$	$mq(\alpha-1)-2\,npk^2\beta$
	$\beta_{21}=1,\ \beta_{33}=n,\ \beta_{42}=q$	$+(mp-nq)ku_4(b,k)=0$

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SUR QUELQUES PROBLÈMES AUX LIMITES DE IV ORDRE

(Résumé)

On considère 48 problèmes aux limites de IV ordre avec un type determiné des conditions ux limites, c'est-à-dire l'équation différentielle

$$y^{IV} + \lambda y = 0, \ \lambda = -k^4 \tag{1}$$

avec les conditions aux limites d'une forme

$$y^{(r)}(a) = y^{(s)}(b) = 0 \tag{2}$$

$$m\, y^{(r_1)}(a) + n\, y^{(s_1)}(b) = p\, y^{(r_2)}(a) + q\, y^{(s_2)}(b) = 0,$$

où r et s prennent les valeurs 0 et 1 (c'est-a-dire les conditions aux limites essentielles), tandis que r_1, r_2; s_1, s_2 les valeurs 0, 1, 2, 3; $y^{(r)}(x) = \dfrac{d^r y}{dx^r}$;
ou

$$(r;\ s;\ r_1\ s_1;\ r_2\ s_2) \qquad \text{(les désignations de Kamke).} \tag{2'}$$

On peut obtenir ces problèmes aux limites (1), (2′) directement du problème (1) et

$$\sum_{\nu=0}^{3} [\alpha_{\mu\nu}\, y^{(\nu)}(a) + \beta_{\mu\nu}\, y^{(\nu)}(b)] = 0, \ \mu = 1, 2, 3, 4 \tag{3}$$

pour les valeurs particulières des paramètres $\alpha_{\mu\nu}$, $\beta_{\mu\nu}$. Ce problème a été considéré dans l'étude [3].

Voici, quelques de ces problèmes aux limites et les équations transendantes pour leur valeurs propres.

Ainsi

pour (0; 0; 11; 22) on obtient $(mq+np)\ (\gamma-\delta)$
$+((mp+nq)\ u_4(b,k)=0,$

pour (0; 0; 11; 32) on obtient $mq(\gamma-\delta)+npk\beta$
$-mpk\,u_2(b,\ k)+nq\,u_3(b,k)=0$

pour (0; 1; 10; 23) on obtient $(mq\,k^2-np)\,\delta+(mq\,k^2+np)\,\gamma$
$-(mp-nq)\,u_2(b,\ k)=0$

pour (0; 1; 22; 33) on obtient $(2\,mq+np)\,\alpha+\ np$
$+(mp+nq)\,u_1(b,\ k)=0$

etc.

Tous les problèmes sont donnés dans les pages 56—63.

Ici

$$\alpha=\cos K\,\mathrm{ch}\,K,\quad \gamma=\cos K\,\mathrm{sh}\,K$$
$$\beta=\sin K\,\mathrm{sh}\,K,\quad \delta=\sin K\,\mathrm{ch}\,K,\quad K=k\,(b-a)$$

et

$$u_1(b,k)=\mathrm{ch}\,K+\cos K,\quad u_3(b,k)=\mathrm{sh}\,K+\sin K.$$
$$u_2(b,k)=\mathrm{ch}\,K-\cos K,\quad u_4(b,k)=\mathrm{sh}\,K-\sin K.$$