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SOME PROPERTIES ON THE FUZZY α–STRUCTURE

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Abstract. In this article we will show that the the class of fuzzy α–sets does not form a fuzzy topology in a fuzzy topological space due to Chang.

The main motivation to write this note is to question the results presented on the article „On fuzzy α–structure and fuzzy structure" by M.Y.Bakier [2].

We recall some notions which are use in the sequel.

Definition 1. [4] *A fuzzy topology on a nonempty set X is a collection of subset $\tau \in I^X$ of such that*

(i) $0, 1 \in \tau$;

(ii) if $\mu, \nu \in \tau$, then $\mu \cap \nu \in \tau$;

(iii) if $\mu_i \in \tau$, $\forall i \in I$, then $\cup_{i \in I} \mu_i \in \tau$.

The pair (X, τ) is called a fuzzy topological space. If $\mu \in \tau$ then μ is said to be fuzzy open set. The closure of a fuzzy set μ (denoted by μ^-), the interior of a fuzzy set μ (denoted by μ^0) and the complement of a fuzzy set μ (denoted by μ^C) of X are defined by

$\mu^- = \cap\{\nu \mid \nu$ *is a fuzzy closed set and* $\mu \leq \nu\}$,

$\mu^0 = \cup\{\lambda \mid \lambda$ *is a fuzzy open set and* $\mu \geq \lambda\}$, ,

$\mu^C(x) = 1 - \mu(x)$, *for all* $x \in X$.

Definition 2. [2] *Let τ be a fuzzy topology on a nonempty set X. A fuzzy set $\mu \in I^X$ is said to be a fuzzy α–set if $\mu \leq \mu^{0-0}$. The class consisting of all fuzzy α–sets is called a fuzzy α–structure and is denoted by τ^α.*

Definition 3. [2] *Let τ be a fuzzy topology on a nonempty set X. A fuzzy set $\mu \in I^X$ is said to be a fuzzy β–set if $\mu \leq \mu^{0-}$. The class consisting of all fuzzy β–sets is a called fuzzy β–structure and is denoted by τ^β.*

In the article „On fuzzy α–structure and fuzzy β–structure" [2] the author claims that a class of fuzzy α–sets forms a fuzzy topology. Unfortunately, the presented result holds in ordinary topology only. We can see from [1] that they rely on the fact, that in any topological space (X, τ) the α–structure τ^α is a topology, larger than τ. Furthermore

$$\tau^\alpha = \{\mu \mid \mu \cap \nu \in \tau^\beta, \text{ for all } \nu \in \tau^\beta\}.$$

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The investigations connected with the topology of $\alpha-$sets in ordinary topology are still active. ([3, 5]). On the other side, since the Singal and Ravjanshi [8] introduced the notion of fuzzy $\alpha-$sets many research papers were done in different directions [6, 7].

We should be careful when we use a fuzzy set instead of an ordinary set. When we work in fuzzy topology the straight generalization of those results is not allowed. In the mentioned article, the author made several mistakes.

In Theorem 3.1 of [2] the author claims that an arbitrary fuzzy topology τ on X consists of exactly those fuzzy sets μ for which $\mu \cap \nu \in \tau^{\beta}$ for all $\nu \in \tau^{\beta}$. Such a statement is incorrect even in ordinary topology, as we can see from the following example. Therefore such a statement is incorrect in a fuzzy topology.

If we read carefully the Theorem 3.1 [2] we can notice that the prove does not correspond to the claim. The author proves that for an arbitrary fuzzy topology τ on X, τ^{α} consists of exactly those fuzzy sets μ for which $\mu \cap \nu \in \tau^{\beta}$ for all $\nu \in \tau^{\beta}$. Even, if he made a typography error in the formulation of the theorem, this claim does not hold in fuzzy topology. In the second part of the proof of the theorem he uses the fact that the fuzzy set $(\mu^{0-0})^C$ and μ^{0-0} are disjoint. Unfortunately, in fuzzy topology a fuzzy set and its complement may be not disjoint.

However, with the next example we will show that in fuzzy topological space the $\alpha-$structure is not a fuzzy topology. Therefore Theorem 3.2 is incorrect. By way, is relies on the description of $\alpha-$structure presented in the Theorem 3.1 which is incorrect also.

Example 1. Let $X = \{a, b, c\}$ A, B amd C be fuzzy sets defined as follows:

$$A(a) = 0.3, \quad A(b) = 0.2, \quad A(c) = 0.7,$$
$$B(a) = 0.8, \quad B(b) = 0.9, \quad B(c) = 0.4,$$
$$C(a) = 0.8, \quad C(b) = 0.9, \quad C(c) = 0.5.$$

Then $\tau = \{0, A, B, A \cap B, A \cup B\}$ is a fuzzy topology on X. By easy computation it can be seen that $C \leq \mathrm{int}(\mathrm{cl}(\mathrm{int}C))$, so C is $\alpha-$set. The fuzzy set A is a fuzzy open set, so A is a fuzzy $\beta-$set. But $C \cap A$ is not a fuzzy $\beta-$set. Therefore the Theorem 3.1 is incorrect.

Since the fuzzy set A is a fuzzy open, so A is a fuzzy $\alpha-$set. We can verify that $A \cap C$ is not a fuzzy $\alpha-$set, so $\alpha-$structure is not a fuzzy topology. It follows that the Theorem 3.2 [2] is incorrect.

From the above discussion it follows that investigations in this direction are not reasonable. We can not call an $\alpha-$structure an $\alpha-$topology. Therefore Theorem 3.3 [2] together with Corollaries 3.1 and 3.2 are erroneous because they treat a $\beta-$structure under the wrong assumption that an $\alpha-$structure is a fuzzy topology.

Therefore the class of fuzzy $\alpha-$sets does not form a fuzzy topology in a fuzzy topological space due to Chang.

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