$$u_{j+2}x_{j,j+1} + \lambda_{j+2}x_{j,j+2} = \lambda_j x_{j,j+2}$$

$$\cdot \quad \cdot \quad \cdot$$

$$u_n x_{j,n-1} + \lambda_n x_{jn} = \lambda_j x_{jn},$$

i.e.

$$u_{j+1}x_{j,j} = (\lambda_j - \lambda_{j+1})x_{j,j+1}$$

$$u_{j+2}x_{j,j+1} = (\lambda_j - \lambda_{j+2})x_{j,j+2}$$

$$\cdot \quad \cdot \quad \cdot$$

$$u_n x_{j,n-1} = (\lambda_j - \lambda_n)x_{j,n}.$$

Multiplying the first k equations of this system, we find $x_{j,j+k}$, $(1 \le k \le n-j)$:

$$x_{j,j+k} = x_{j,j} \cdot \frac{u_{j+1}u_{j+2}\cdots u_{j+k}}{(\lambda_j - \lambda_{j+1})\cdots(\lambda_j - \lambda_{j+k})} =$$

$$= x_{j,j} \cdot \prod_{v=1}^{k} \frac{(n+1-j-v)(j+v)}{v(z+2j+1-v)} = x_{j,j} \cdot \frac{\binom{j+k}{j}\binom{n+j+1+z}{n-j-k}}{\binom{n+j+1+z}{n-j}}.$$

Hence the s-th component of the vector $\mathbf{X}_j$ is

$$x_{js} = \binom{s}{j}\binom{n+j+1+z}{n-s}, \qquad (0 \le s \le n)$$

where $\binom{s}{j} = 0$ if $s < j$.

It is easy to verify that the inverse matrix of the matrix $B = [b_{ij}] = \left[\binom{j}{i}\right]$ is given by $(B^{-1})_{ij} = (-1)^{i+j}\binom{j}{i}$. Hence for the required eigenvector $\mathbf{Y}_j$ of the matrix A we obtain

$$\mathbf{Y}_j = B^{-1}\mathbf{X}_j \qquad (0 \le j \le n)$$

where

$$Y_j = \Big[y_{j0}, y_{j1}, \cdots, y_{jn}\Big]^T, \qquad (0 \le j \le n) \tag{2}$$

with coordinates

$$y_{jr} = \sum_{s=0}^{n} (B^{-1})_{rs} x_{js},$$

i.e.

$$y_{jr} = \sum_{s=j}^{n} (-1)^{r+s}\binom{s}{r}\binom{s}{j}\binom{n+j+1+z}{n-s}, \qquad (0 \le j, r \le n). \tag{3}$$

Now the general solution of the system (1) is given by