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EQUIVALENCE OF INTRINSIC SHAPE, BASED ON $\mathcal{V}$ -CONTINUOUS FUNCTIONS, AND SHAPE

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Abstract. In this paper is given a direct proof that the intrinsic shape category $InSh$ constructed with continuous functions over coverings, is equivalent to original shape category Sh of Borsuk obtained by embedding compact metric spaces in Hilbert cube Q. The functor $Sh \to InSh$ is established taking a fundamental sequence $(\bar{f}_n)$ from X to Y in the sense of Borsuk, and by associating to the continuous function $\bar{f}_n : Q \to Q$ mapping some neighborhood of X into a union of the members of a covering $\mathcal{V}$ of Y, a $\mathcal{V}$ - continuous function $f_n : X \to Y$, and forming the proximate sequence (f_n) in the sense of N. Shekutkovski, Top. Proc. 39 (2012).

1. Introduction

For compact metric spaces, shape theory was introduced by K. Borsuk. His oriiginal approach was by embedding a compact metric space in Hilbert cube. Further on we will denote his shape category of compact metric spaces by Sh.

From the early beginning of the theory arised the question of intrinsic definition of shape i.e., a definition without external spaces like Hilbert cube. A shape category HN is obtained by intrinsic definition by Sanjurjo in [6], and it is shown that HN and Sh are equivalent constructing a functor $HN \to Sh$. A shape category, also by intrinsic approach is obtained by Shekutkovski in [7], which we will further on denote by $InSh$. Using the result [3] and [4] about equivalence of categories $InSh$ and HN, and the isomorphic functor $Sh \to HN$ the equivalence categories Sh and $InSh$ is indirectly proven in [5], using.

In this paper we will give a direct proof of equivalence of categories Sh and $InSh$ constructing a functor $Sh \to InSh$, i.e. compared with the functor from [6], it is in the opposite direction. The construction requires some new ideas, among them the introduced notions of depth and regular covering.

Let X and Y be compact metric spaces. By a covering we understand a covering consisting of open sets. We repeat the intrinsic approach to shape fron [7] (also [9]):

Definition 1. Suppose $\mathcal{V}$ is a finite covering of Y. A function $f : X \to Y$ is **$\mathcal{V}$-continuous at point** $x \in X$, if there exists a neighborhood U_x of x, and $V \in \mathcal{V}$, such that $f(U_x) \subseteq V$.

A function $f : X \to Y$ is **$\mathcal{V}$-continuous**, if it is $\mathcal{V}$-continuous at every point $x \in X$.

In this case, the family of all neighborhoods U_x, form a covering of X. By this, $f : X \to Y$ is $\mathcal{V}$-continuous if there exists a finite covering $\mathcal{U}$ of X, such that for any $U \in \mathcal{U}$, there exists $V \in \mathcal{V}$ such that $f(U) \subseteq V$. We denote shortly: there exists $\mathcal{V}$, such that $f(\mathcal{U}) \prec \mathcal{V}$.

If $f : X \to Y$ is $\mathcal{V}$-continuous, then $f : X \to Y$ is $\mathcal{W}$-continuous for any $\mathcal{W}$, such that $\mathcal{V} \prec \mathcal{W}$.

If $\mathcal{V}$ is a finite covering of Y, and $V \in \mathcal{V}$, than star of V is the open set $st(V) = \{W | W \in \mathcal{V},\ W \cap V \neq \varnothing\}$. We form a new covering $st(\mathcal{V}) = \{st(V) | V \in \mathcal{V}\}$.

Definition 2. The functions $f, g : X \to Y$ are **$\mathcal{V}$-homotopic**, if there exists a function $F : X \times I \to Y$ such that:

1) F is $st(\mathcal{V})$-continuous,

2) F is $\mathcal{V}$-continuous at all points of $X \times \partial I$, and

3) $F(x, 0) = f(x)$, $F(x, 1) = g(x)$.

The relation of $\mathcal{V}$-homotopy is denoted by $f \underset{\mathcal{V}}{\sim} g$. This is an equivalence relation.

Ussualy, the condition 2) of the previous statement is formulated as:

2) there exists an neighbourhood N of $\partial I = \{0, 1\}$ such that $F|_{X \times N}$ is $\mathcal{V}$-continuous.

Definition 3. A **cofinal sequence of finite coverings** $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots \mathcal{V}_n \succ \ldots$ is a sequence of finite coverings of spaces, such that for any covering $\mathcal{V}$, there exists n, such that $\mathcal{V}_n \prec \mathcal{V}$.

In a compact metric space there exists such a sequence. This fact allows working with proximate sequences instead with proximate nets.

Definition 4. The sequence (f_n) of functions $f_n : X \to Y$ is a **proximate sequence** from X to Y, if there exists a cofinal sequence of finite coverings of Y, $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots \mathcal{V}_n \succ \ldots$, and for all indexes f_n and f_{n+1} are $\mathcal{V}_n$-homotopic.

In this case we say that (f_n) is a proximate sequence over $(\mathcal{V}_n)$.

If (f_n) and (f'_n) are proximate sequences from X to Y, than there exists a cofinal sequence of finite coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots \mathcal{V}_n \succ \ldots$ such that (f_n) and (f'_n) are proximate sequences over $(\mathcal{V}_n)$.

Definition 5. Two proximate sequences (f_n) and (f'_n) are **homotopic** if there exists a cofinal sequence of finite coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots \mathcal{V}_n \succ \ldots$ of Y, such that (f_n) and (f'_n) are $\mathcal{V}_n$-homotopic for all integers n.

We say that (f_n) and (f'_n) are homotopic over $(\mathcal{V}_n)$.

Let $(f_n) : X \to Y$ be a proximate sequence over $(\mathcal{V}_n)$ and $(g_k) : Y \to Z$ be a proximate sequence over $(\mathcal{W}_k)$. For a covering $\mathcal{W}_k$ of Z, there exists a covering $\mathcal{V}_{n_k}$ of Y such that $g\left(\mathcal{V}_{n_k}\right) \prec \mathcal{W}_k$. Then, the composition is the proximate sequence $(h_k) = (g_k f_{n_k}) : X \to Z$. In [7] is proven that compact metric spaces and homotopy classes of proximate sequences $[(f_n)]$ form the shape category $InSh$ i.e. isomorphic spaces in this category has the same shape.

We repeat the original definition of Bosuk of shape categry Sh. Let X, Y and Z, be compact metric spaces, embedded in the Hilbert space Q.

A sequence of maps $f_k : Q \to Q$, $k = 1, 2, 3, ...$, is **fundamental sequence from X to Y**, if for every neighborhood V of Y, there exsist a neighborhood U of X and there exists $k_0 \in \mathbb{N}$, such that $f_k|_U \simeq f_{k+1}|_U$ in V for all $k \geq k_0$. A fundamental sequence is denoted with $(f_k : X \to Y)_{Q,Q}$.

Proposition 1. If $(f_k : X \to Y)_{Q,Q}$ is a fundamental sequence, then there exists a decreasing sequence of neighborhoods of Y, $V_1 \supseteq V_2 \supseteq ...$ such that $\cap V_n = Y$, and there exists a decreasing sequence of neighborhoods of

X, $U_1 \supseteq U_2 \supseteq ...$ such that $\cap U_n = X$ and such that for all integers, $f_k|_{U_k} \simeq f_{k+1}|_{U_k}$ in V_k.

Two fundamental sequences $(f_k : X \to Y)_{Q,Q}$ and $(f'_k : X \to Y)_{Q,Q}$ are **homotopic**, if for every neighborhood V of Y in Q, there exist neighborhood U of X in Q and $k_0 \in \mathbb{N}$, such that f_k is homotopic to f'_k in V, for all $k \geq k_0$.

The relation of homotopy $(f_k : X \to Y) \simeq (f'_k : X \to Y)$ of fundamental sequences is an equivalence relation. We use symbol [ ] to denote homotopy classes.

The composition of fundamental sequences $(f_k : X \to Y)$ and $(g_k : Y \to Z)$, is the fundamental sequence $(g_k f_k : X \to Z)$. The composition of classes

$$[(f_k : X \to Y)] \text{ and } [(g_k : Y \to Z)]$$

is the class $[(g_k f_k : X \to Z)]$.

2. Equivalence of categories

Let X be a set and $\mathcal{V} = \{V_i \,| i = 1, 2, ..., n\}$ be a finite set of subsets of X. If $V \in \mathcal{V}$, we define **depth** of V in $\mathcal{V}$, to be the biggest number $k \in \mathbb{N}$ such that there exist sequence of elements of $\mathcal{V}$ such that $V \subset V_2 \subset V_3 \subset ... \subset V_k$. (if V is not a proper subset of any element in $\mathcal{V}$ then depth of V is 1). The depth of V we denote with $depth\,(V)$.

A covering $\mathcal{V}$ of Y in X is **regular** if it satisfies the following conditions:

1) If $V \in \mathcal{V}$ than $V \cap Y \neq \emptyset$.

2) If $U, V \in \mathcal{V}$ and $U \cap V \neq \emptyset$, than $U \cap V \in \mathcal{V}$.

About the condition 1) see definition of proper covering, ([10] , Definition 8.1., p. 249), while the condition 2) together with 1) shows that $\mathcal{V}$ is a regular family relative to Y in the sense of [10] (Definition 3.5. p. 262).

For a covering $\mathcal{V}$ we introduce the notation $|\mathcal{V}| = \cup \{V \,| V \in \mathcal{V}\}$

We define a function $r_{\mathcal{V}} : |\mathcal{V}| \to Y$ in the following way:

Suppose n is the biggest depth of the elements of $\mathcal{V}$. A function $r_{\mathcal{V}} : |\mathcal{V}| \to Y$ will be defined by induction.

For points y belonging to $V \in \mathcal{V}$, such that $depth\,(V) = n$, we choose a fixed point $[V] \in V \cap Y$ and put $r_{\mathcal{V}}\,(y) = [V]$.

Suppose the function is defined for all y belonging to some $V \in \mathcal{V}$, such that $depth\,(V) > n - k$ for some natural number k.

If y belongs to some $V \in \mathcal{V}$ with $depth\,(V) = n - k$, and $r_{\mathcal{V}}\,(y)$ is not defined yet, i.e. $y \in V \backslash \cup \{V\,|V \in \mathcal{V},\, depth\,(V) > n - k\}$, we choose a fixed point $[V]$, $[V] \in (V \backslash \cup \{V\,|V \in \mathcal{V},\, depth\,(V) > n - k\}) \cap Y$ and put $r_{\mathcal{V}}\,(y) = [V]$.

The function $r_{\mathcal{V}}$ is well defined and is $\mathcal{V}$ -continuous.

In fact, $r_{\mathcal{V}}\,(y) = [V]$ if and only if V is the smallest set in $\mathcal{V}$ which contains y i.e $V = \bigcap\limits_{\substack{U \in \mathcal{V} \\ y \in U}} U$.

Now, if $\mathcal{V}$ is a regular covering of Y and $\bar{f} : X \to |\mathcal{V}|$ is a continuous function, we define function $f : X \to Y$ with $f\,(x) = r_{\mathcal{V}} \bar{f}\,(x)$ for all $x \in X$.

The function f is well defined and since $\bar{f}$ is continuous, the function f is $\mathcal{V}$-continuous.

We will say that the function is f is obtained from a continuous function $\bar{f}$ and covering $\mathcal{V}$.

Theorem 1. If Y is compact metric space embedded in Hilbert cube Q, $\mathcal{V}$ and $\mathcal{W}$ are regular coverings of Y in Q such that $\mathcal{W} \prec \mathcal{V}$, then $r_{\mathcal{V}} : |\mathcal{W}| \to Y$ (the restriction of $r_{\mathcal{V}}$ to $|\mathcal{W}|$) and $r_{\mathcal{W}}$ are $\mathcal{V}$ -homotopic.

Proof. We consider the function $R : |\mathcal{W}| \times I \to Y$ defined by

$$R\,(x,t) = \begin{cases} r_{\mathcal{V}}\,(x)\,, & (x,t) \in |\mathcal{W}| \times [0,1) \\ r_{\mathcal{W}}\,(x)\,, & (x,1) \in |\mathcal{W}| \times \{1\} \end{cases}$$

If $(x,t) \in |\mathcal{W}| \times [0,1)$, then $R\,(x,t) = r_{\mathcal{V}}\,(x)$, and R is $\mathcal{V}$-continuous in (x,t).

If $(x,1) \in |\mathcal{W}| \times \{1\}$, then $R\,(x,1) = r_{\mathcal{W}}\,(x) = [W]$, where W is the smallest set in $\mathcal{W}$ that contains x.

From $\mathcal{W} \prec \mathcal{V}$, it follows that $W \subseteq V \in \mathcal{V}$, and we can choose V to be the smallest set in $\mathcal{V}$, with the property $W \subseteq V$. Then $r_{\mathcal{V}}\,(V) \in V \cap Y$ and

$$R\,(W \times 1) = r_{\mathcal{W}}\,(W) \in W \cap Y \subset V \cap Y. \qquad (*)$$

We take the neighborhood $W \times [0,1]$ of $(x,1)$ and $(w,t) \in W \times [0,1)$. There is a smallest set V_w in $\mathcal{V}$ such that $w \in V_w$. We obtain

$$R\,(w,t) = r_{\mathcal{V}}\,(w) = [V_w] \in V_w \cap Y, \text{ for all } t \in [0,1). \qquad (**)$$

From the construction $V_w \subseteq V$ for all $w \in W$. Finally from (*) and (**),

$$R\,(W \times [0,1]) = R\,(W \times [0,1)) \cup R\,(W \times 1) \subseteq V \cap Y.$$

It follows that R is $\mathcal{V}$-continuous at $(x,1) \in |\mathcal{W}| \times \{1\}$, and $R(x,0) = r_{\mathcal{V}}(x)$, $R(x,1) = r_{\mathcal{W}}(x)$.

By Proposition 1, if $(\bar{f}_n)$ is fundamental sequence from X to Y, there exists $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \dots$, a cofinal sequence of finite regular coverings of Y in Q, and there exists a cofinal sequence of finite regular coverings of X in Q, $\mathcal{U}_1 \succ \mathcal{U}_2 \succ \dots$ such that $\bar{f}_n(|\mathcal{U}_n|) \prec |\mathcal{V}_n|$, and continuous functions $\bar{f}_n$ and $\bar{f}_{n+1}$, are homotopic in $|\mathcal{V}_n|$. □

We define a function $f_n : X \to Y$, $n \in \mathbb{N}$ by $f_n(x) = r_{\mathcal{V}_n}\bar{f}_n(x)$ for $x \in X$.

Theorem 2. 1) If $(\bar{f}_n)$ is fundamental sequence then (f_n) is proximate sequence.

2) If two fundamental sequences $(\bar{f}_n)$ and $(\bar{f}'_n)$ are homotopic, then the obtained from them proximate sequences (f_n) and (f'_n) are homotopic.

Proof. 1) Suppose $\overline{f}_{n,n+1}$, is the homotopy connecting $\overline{f}_n$ and $\overline{f}_{n+1}$. We define $f_{n,n+1} : X \times I \to Y$ by

$$f_{n,n+1}(x,t) = r_{\mathcal{V}_n}\overline{f}_{n,n+1}(x,t).$$

Then $f_{n,n+1}$ is $\mathcal{V}_n$ -continuous and

$$f_{n,n+1}(x,0) = r_{\mathcal{V}_n}\overline{f}_n(x), \; f_{n,n+1}(x,1) = r_{\mathcal{V}_n}\overline{f}_{n+1}(x). \qquad (*)$$

By the previous theorem $r_{\mathcal{V}_n}\big|_{|\mathcal{V}_{n+1}|}$ (the restriction of $r_{\mathcal{V}_n}$ to $|\mathcal{W}|$) and $r_{\mathcal{V}_{n+1}}$ are $\mathcal{V}_n$-homotopic, by a homotopy $R : |\mathcal{V}_{n+1}| \times I \to Y$ i.e.

$$R(x,0) = r_{\mathcal{V}_n}(x), \; R(x,1) = r_{\mathcal{V}_{n+1}}(x).$$

Then the $\mathcal{V}_n$-homotopy $R\overline{f}_{n+1} : X \times I \to Y$ satisfies

$$R\overline{f}_{n+1}(x,0) = r_{\mathcal{V}_n}\overline{f}_{n+1}(x), \; R\overline{f}_{n+1}(x,1) = r_{\mathcal{V}_{n+1}}\overline{f}_{n+1}(x) = f_{n+1}(x). \qquad (**)$$

Since $\mathcal{V}_n$-homotopy is an equivalence relation by (*) and (**) it follows that $r_{\mathcal{V}_n}\overline{f}_n(x) = f_n(x)$ and $r_{\mathcal{V}_{n+1}}\overline{f}_{n+1}(x) = f_{n+1}(x)$ are $\mathcal{V}_n$-homotopic.

2) Suppose $\overline{F_n}$, is the homotopy connecting $\overline{f}_n$ and $\overline{f}'_n$. We define $F_n : X \times I \to Y$ by $F_n(x,t) = r_{\mathcal{V}_n}\overline{F_n}(x,t)$.

Then F_n is $\mathcal{V}_n$ -continuous and $st(\mathcal{V}_n)$ continuous at all points of $X \times \partial I$, and $F_n(x,0) = f_n(x)$, $F_n(x,1) = f'_n(x)$. □

We will describe a functor $\Phi : Sh \to InSh$.

1) On compact metric spaces is defined by $\Phi(X) = X$, for every compact metric space X.

2) and is defined with $\Phi\left(\left[\left(\bar{f}_n\right)\right]\right) = [(f_n)]$ for every class of fundamental sequences $\left[\left(\bar{f}_n\right)\right]$ from X to Y.

Theorem 3. $\Phi : Sh \to InSh$ is a functor which is isomorphism of categories.

Proof. First we will prove that for two fundamental sequences $(f_n) : X \to Y$ and $(g_n) : Y \to X$ holds

$$\Phi\left(\left[(\bar{g}_n)\left(\bar{f}_n\right)\right]\right) = \Phi\left([(\bar{g}_n)]\right)\Phi\left(\left[\left(\bar{f}_n\right)\right]\right).$$

As in the beginning of this section there exists a cofinal sequence of finite regular coverings $\mathcal{W}_1 \succ \mathcal{W}_2 \succ \ldots$ of Z in Q, there exists a cofinal sequence of finite regular coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots$ of Y in Q, and there exists a cofinal sequence of finite regular coverings $\mathcal{U}_1 \succ \mathcal{U}_2 \succ \ldots$ of X in Q, such that $\bar{f}_n(|\mathcal{U}_n|) \subseteq |\mathcal{V}_n|$ and $\bar{f}_n(|\mathcal{V}_n|) \subseteq |\mathcal{W}_n|$, and such that continuous functions $\bar{f}_n$, $\bar{f}_{n+1}$, are homotopic in $|\mathcal{V}_n|$ and $\overline{g}_n$, $\overline{g}_{n+1}$, are homotopic in $|\mathcal{W}_n|$ for all n.

Suppose a proximate sequence (g_n) from Y to Z is obtained from fundamental sequence $(\bar{g}_n)$, taking a cofinal sequence of finite regular coverings $(\mathcal{W}_k)$ of Z in Q.

Suppose (f_{n_k}) from X to Y is a proximate subsequence of the proximate sequence (f_n) obtained from fundamental sequence $\left(\bar{f}_n\right)$. The subsequence of natural numbers is chosen such that $f_{n_k}\left(\mathcal{V}_{n_k}\right) \prec \mathcal{W}_k$.

The fundamental sequences $\left(\bar{f}_n\right)$ and $\left(\bar{f}_{n_k}\right)$ are in the same class. By theorem from [7], (f_{n_k}) and (f_n) are in the same class and if we put $\bar{g}_k\bar{f}_{n_k} = \overline{h}_k$.

In fact we have to prove

$$\left[r_{\mathcal{W}_k}\overline{h}_k\right] = \left[\left(r_{\mathcal{W}_k}\bar{g}_k\right)\left(r_{\mathcal{V}_{n\,k}}\bar{f}_{n_k}\right)\right]$$

or that $r_{\mathcal{W}_k}\overline{h}_k$ and $\left(r_{\mathcal{W}_k}\bar{g}_k\right)\left(r_{\mathcal{V}_{n\,k}}\bar{f}_{n_k}\right)$ are homotopic.

Take a point x in X. By definition

$$r_{W_k}\overline{h}_k(x) = [W] \qquad (*)$$

where W is the smallest set W in $\mathcal{W}_k$ such that $\bar{g}_k\bar{f}_{n_k}\left(x\right) = \overline{h}_k\left(x\right) \in W$.

Since $\bar{g}_k\left(\mathcal{V}_{n_k}\right) \prec \mathcal{W}_k$ there exist V' in $\mathcal{V}$ such that $\bar{g}_k\left(V'\right) \subseteq W$.

On the other hand, by definition $r_{\mathcal{V}_{n_k}}\bar{f}_{n_k}\left(x\right) = [V]$ where V is the smallest set V in $\mathcal{V}_{n_k}$ such that $\bar{f}_{n_k}(x) \in V$.

Since V is the smallest, then $V \subseteq V'$ and it follows $\bar{g}_k(V) \subseteq W$. Then

$$\left(r_{\mathcal{W}_k}\bar{g}_k\right)\left(r_{\mathcal{V}_{n_k}}\bar{f}_{n_k}\right)(x) = r_{\mathcal{W}_k}\left(\bar{g}_k[V]\right) \in W \qquad (**)$$

Then, by (**) and (*) $r_{\mathcal{W}_k}\overline{h}_k = h_k$ and $\left(r_{\mathcal{W}_k}\bar{g}_k\right)\left(r_{\mathcal{V}_{n_k}}\bar{f}_{n_k}\right) = g_k f_{n_k}$ are $\mathcal{W}_k$-near, and since h_k is $\mathcal{W}_k$-continuous, by Lemma 1.1 from [14] we have that h_k and $g_k f_{n_k}$ are $\mathcal{W}_k$- homotopic.

Now, we will prove that

$$\Phi\left([(1_X)]\right) = 1_{\Phi(X)}.$$

One represent of the identical morphism in Sh is the class of fundamental sequences from X to X is $\left(\overline{1}_n\right)$, where $\overline{1}_n : Q \to Q$, $n \in \mathbb{N}$ are copies of identical map defined by $\overline{1}_n(x) = x$, $x \in X$.

Then $\Phi\left(\left[\left(\overline{1}_n\right)\right]\right) = [(1_n)]$, where $1_n : X \to X$, $n \in \mathbb{N}$ are copies of identical map.

$[(1_n)]$ is the identical morphism in $InSh$ since for proximate sequences $(f_n) : X \to Y$ and $(g_n) : Y \to X$ holds $(f_n)(1_n) = (f_n)$ and $(g_n)(1_n) = (g_n)$.

It follows $\Phi\left([(1_X)]\right) = 1_{\Phi(X)}$.

To prove that $\Phi : Sh \to InSh$ is a functor which is an isomorphism of categories we use the following reformulation of theorem 1 of [5] : For every proximate sequence $(g_n) : X \to Y$ there exists a fundamental sequence $\left(\bar{f}_n\right)$ from X to Y and a cofinal sequence of coverings $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \ldots$, such that for such that $\bar{f}_n|_X$ and g_n are $\mathcal{V}_n-$ close for all integers. Also, all fundamental sequences obtained from (f_n) in this way are homotopic.

One proximate sequence (f_n) obtained from fundamental sequence $\left(\bar{f}_n\right)$, it consists of $\mathcal{V}_n-$ close functions f_n and $\bar{f}_n$. Therefore (f_n) and (g_n) are homotopic and it follows that the functor is **surjective**.

To prove that the functor is **injective**, suppose the proximate sequences (f_n) and (f'_n) from X to Y are obtained from fundamental sequences $\left(\bar{f}_n\right)$ and $\left(\bar{f}'_n\right)$ from X to Y, respectively. Suppose (f_n) and (f'_n) are homotopic, i.e. f_n and f'_n are connected by homotopy F_n for all positive integers. By Ho‘s theorem, in fact from its form Lemma 1 from [6], it follows that there exists a continuous homotopy $\bar{F}_n$ connecting $\bar{f}_n$ and $\bar{f}'_n$ for all natural numbers n. □

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ЕКВИВАЛЕНЦИЈА НА ВНАТРЕШЕН ОБЛИК, БАЗИРАН НА $\mathcal{V}-$ НЕПРЕКИНАТИ ФУНКЦИИ, И ОБЛИК

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Р е з и м е

Во овој труд даден е директен доказ дека категоријата на внатрешен облик $InSh$ конструирана со непрекинати функции над покривачи е еквивалентна со оригиналната категорија на облик Sh на Борсук добиена со вложување на компактни метрички простори во Хилбертовиот куб Q. Функторот $Sh \to InSh$ е добиен земајќи фундаментална низа $\left(\bar{f}_n\right)$ од X во Y во смисла на Борсук и на непрекинатата функција $\bar{f}_n : Q \to Q$ која пресликува некоја околина на X во унија на членови на покривач $\mathcal{V}$ на Y, и се придружува $\mathcal{V}$ - непрекината функција $f_n : X \to Y$, и формирајќи проксимативната низа (f_n) во смисла на N. Shekutkovski, Top. Proc. 39 (2012).

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