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NONLINEAR CONTRACTIONS AND FIXED POINTS IN COMPLETE DISLOCATED AND b-DISLOCATED METRIC SPACES

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Abstract. In this paper, we continue the study of complete dislocated and b-dislocated metric spaces and established some common fixed point theorems for one and two mappings. Our results generalizes and extend some existing results in the literature in a class effectively larger such as b-dislocated metric spaces, where the self distance for a point may not be equal to zero.

1. INTRODUCTION

The concept of b-metric space was introduced by Bakhtin [4] and extensively used by Czerwik in [10]. After that, several interesting results about the existence of a fixed point for single-valued and multi-valued operators in b-metric spaces have been obtained. Recently there are a number of generalizations of metric space. Some of them are the notions of dislocated metric spaces and b-dislocated metric spaces where the distance of a point in the self may not be zero. These spaces was introduced and studied by Hitzler and Seda [5], Nawab Hussain et.al [7]. Also in [7] are presented some topological aspects and properties of b-dislocated metrics. Subsequently, several authors have studied the problem of existence and uniqueness of a fixed point for single-valued and set-valued mappings and different types of contractions in these spaces.

The purpose of this paper is to unify and generalize some recent results in the setting of dislocated and b-dislocated metric spaces using a class of continuous functions G_4.

2. PRELIMINARIES

Definition 2.1 [6]. Let X be a nonempty set and a mapping $d_l : X \times X \to [0,\infty)$ is called a *dislocated metric* (or simply d_l-metric) if the following conditions hold for any $x, y, z \in X$:

i. If $d_l(x,y) = 0$, then $x = y$
ii. $d_l(x,y) = d_l(y,x)$

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iii. $d_l(x,y) \le d_l(x,z) + d_l(z,y)$

The pair (X,d_l) is called a *dislocated metric space* (or d -metric space for short). Note that when $x = y$, $d_l(x,y)$ may not be 0 .

Example 2.2. If $X = R$, then $d(x,y) = |x| + |y|$ defines a dislocated metric on X .

Definition 2.3 [6]. A sequence (x_n) in d_l -metric space (X,d_l) is called:

(1) a *Cauchy sequence* if, for given $\varepsilon > 0$, there exists $n_0 \in N$ such that for all $m,n \ge n_0$, we have $d_l(x_m,x_n) < \varepsilon$ or $\lim_{n,m\to\infty} d_l(x_n,x_m) = 0$,

(2) *convergent* with respect to d_l if there exists $x \in X$ such that $d_l(x_n,x) \to 0$ as $n \to \infty$. In this case, x is called the limit of (x_n) and we write $x_n \to x$.

A d_l -metric space X is called *complete* if every Cauchy sequence in X converges to a point in X .

Definition 2.4[8]. Let X be a nonempty set and a mapping $b_d : X \times X \to [0,\infty)$ is called a b *-dislocated metric* (or simply b_d *-dislocated metric*) if the following conditions hold for any $x,y,z \in X$ and $s \ge 1$:

a. If $b_d(x,y) = 0$, then $x = y$,

b. $b_d(x,y) = b_d(y,x)$,

c. $b_d(x,y) \le s[b_d(x,z) + b_d(z,y)]$.

The pair (X,b_d) is called a b -dislocated metric space. And the class of b -dislocated metric space is larger than that of dislocated metric spaces, since a b -dislocated metric is a dislocated metric when $s = 1$.

In [8] was showed that each b_d -metric on X generates a topology τ_{b_d} whose base is the family of open b_d -balls $B_{b_d}(x,\varepsilon) = \{y \in X : b_d(x,y) < \varepsilon\}$

Also in [8] are presented some topological properties of b_d -metric spaces

Definition 2.5. Let (X,b_d) be a b_d -metric space, and (x_n) be a sequence of points in X . A point $x \in X$ is said to be the limit of the sequence (x_n) if $\lim_{n\to\infty} b_d(x_n,x) = 0$ and we say that the sequence (x_n) is b_d -convergent to x and denote it by $x_n \to x$ as $n \to \infty$.

The limit of a b_d -convergent sequence in a b_d -metric space is unique [8, Proposition 1.27].

Definition 2.6. A sequence (x_n) in a b_d -metric space (X,b_d) is called a b_d -Cauchy sequence iff, given $\varepsilon > 0$, there exists $n_0 \in N$ such that for all $n,m > n_0$, we have $b_d(x_n,x_m) < \varepsilon$ or $\lim_{n,m\to\infty} b_d(x_n,x_m) = 0$. Every b_d -convergent sequence in a b_d -metric space is a b_d -Cauchy sequence.

Remark 2.7. The sequence (x_n) in a b_d -metric space (X,b_d) is called a b_d -Cauchy sequence iff $\lim_{n,m\to\infty} b_d(x_n,x_{n+p}) = 0$ for all $p \in N^*$

Definition 2.8. A b_d -metric space (X,b_d) is called complete if every b_d -Cauchy sequence in X is b_d -convergent.
In general a b_d -metric is not continuous, as in Example 1.31 in [8] showed.

Example 2.9. Let $X = R^+ \cup \{0\}$ and any constant $\alpha > 0$. Define the function $d_l : X \times X \to [0,\infty)$ by $d_l(x,y) = \alpha(x+y)$. Then, the pair (X,d_l) is a dislocated metric space.

Lemma 2.10. Let (X,b_d) be a b -dislocated metric space with parameter $s \geq 1$. Suppose that (x_n) and (y_n) are b_d -convergent to $x, y \in X$, respectively. Then we have

$$\frac{1}{s^2} b_d(x,y) \leq \lim_{n\to\infty} \inf b_d(x_n,y_n) \leq \lim_{n\to\infty} \sup b_d(x_n,y_n) \leq s^2 b_d(x,y)$$

In particular, if $b_d(x,y) = 0$, then we have $\lim_{n\to\infty} b_d(x_n,y_n) = 0 = b_d(x,y)$. Moreover, for each $z \in X$, we have

$$\frac{1}{s} b_d(x,z) \leq \lim_{n\to\infty} \inf b_d(x_n,z) \leq \lim_{n\to\infty} \sup b_d(x_n,z) \leq s b_d(x,z)$$

In particular, if $b_d(x,z) = 0$, then we have $\lim_{n\to\infty} b_d(x_n,z) = 0 = b_d(x,z)$.

Some examples in the literature shows that in general a b -dislocated metric is not continuous.

Example 2.11. If $X = R^+ \cup \{0\}$, then $b_d(x,y) = (x+y)^2$ defines a b -dislocated metric on X with parameter $s = 2$.

3. MAIN RESULT

We consider the set G_4 of all continuous functions $g : [0,\infty)^4 \to [0,\infty)$ with the following properties:

a) g is non-decreasing in respect to each variable

b) $g(t,t,t,t) \le t, t \in [0,\infty)$

Some examples of these functions are as follows:

$$g_1 : g(t_1,t_2,t_3,t_4) = \max\{t_1,t_2,t_3,t_4\}$$
$$g_2 : g(t_1,t_2,t_3,t_4) = \max\{t_1+t_2, t_2+t_3, t_1+t_3, t_3+t_4\}$$
$$g_3 : g(t_1,t_2,t_3,t_4) = [\max\{t_1t_2, t_2t_3, t_3t_1, t_3t_4\}]^{\frac{1}{2}}$$
$$g_4 : g(t_1,t_2,t_3,t_4) = [\max\{t_1{}^p, t_2{}^p, t_3{}^p, t_4{}^p\}]^{\frac{1}{p}},\ p > 0.$$

Theorem 3.1. Let (X,d) be a complete b-dislocated metric space with parameter $s \ge 1$ and $T,S : X \to X$ two mappings satisfying the following contractive condition

$$sd(Sx,Ty) \le c\, g[d(x,y), d(x,Sx), d(y,Ty), \tfrac{d(x,Sx)d(y,Ty)}{1+d(x,y)}] \tag{1}$$

for all $x, y \in X$ where $g \in G_4$ and $0 \le c < 1$. Then T and S have a unique common fixed point and if u is a common fixed point of S and T, then $d(u,u) = 0$.

Proof. Let x_0 be an arbitrary point in X. Define the sequence (x_n) as follows:

$$x_1 = S(x_0), x_2 = T(x_1), ..., x_{2n} = T(x_{2n-1}), x_{2n+1} = S(x_{2n}), ...$$

if we assume that for some $n \in N$, $x_{2n+1} = x_{2n}$ then $x_{2n} = x_{2n+1} = Sx_{2n}$ and also using the contractive condition of theorem we will have that $x_{2n+1} = x_{2n}$ is a fixed point of T . Thus we assume that for $n \in N$, $x_{2n+1} \ne x_{2n}$. By condition (1) we have:

$$\begin{aligned} sd(x_{2n+1}, x_{2n+2}) &= sd(Sx_{2n}, Tx_{2n+1}) \\ &\le cg[d(x_{2n}, x_{2n+1}), d(x_{2n}, Sx_{2n}), d(x_{2n+1}, Tx_{2n+1}), \tfrac{d(x_{2n},Sx_{2n})d(x_{2n+1},Tx_{2n+1})}{1+d(x_{2n},x_{2n+1})}] \\ &= cg[d(x_{2n}, x_{2n+1}), d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2}), \tfrac{d(x_{2n},x_{2n+1})d(x_{2n+1},x_{2n+2})}{1+d(x_{2n},x_{2n+1})}] \\ &\le cd(x_{2n+1}, x_{2n}). \end{aligned}$$

Thus

$$d(x_{2n+1}, x_{2n+2}) \le \tfrac{c}{s} d(x_{2n}, x_{2n+1}) \tag{2}$$

Similarly by condition (1) have:

$$\begin{aligned} sd(x_{2n}, x_{2n+1}) &= sd(Tx_{2n-1}, Sx_{2n}) \\ &= sd(Sx_{2n}, Tx_{2n-1}) \\ &\le cg[d(x_{2n}, x_{2n-1}), d(x_{2n}, Sx_{2n}), d(x_{2n-1}, Tx_{2n-1}), \tfrac{d(x_{2n},Sx_{2n})d(x_{2n-1},Tx_{2n-1})}{1+d(x_{2n},x_{2n-1})}] \\ &= cg[d(x_{2n}, x_{2n-1}), d(x_{2n}, x_{2n+1}), d(x_{2n-1}, x_{2n}), \tfrac{d(x_{2n},x_{2n+1})d(x_{2n-1},x_{2n})}{1+d(x_{2n},x_{2n-1})}] \\ &\le cd(x_{2n-1}, x_{2n}). \end{aligned}$$

Thus

$$d(x_{2n}, x_{2n+1}) \le \tfrac{c}{s} d(x_{2n-1}, x_{2n})\ . \tag{3}$$

Generally by conditions (2), (3) and denoting $k=\frac{c}{s}$, we have

$$d(x_{2n+1},x_{2n+2})\le kd(x_{2n},x_{2n+1})\le ...\le k^{2n}d(x_0,x_1) \text{ for } n\in\mathbb{N}.$$

Since $0\le k<1$, taking limit for $n\to\infty$ we have

$$d(x_{2n+1},x_{2n+2})\to 0. \tag{4}$$

Now, we prove that (x_n) is a b_d -Cauchy sequence, and to do this let be $m,n>0$ with $m>n$, and using definition 2.4 (c) we have

$$\begin{aligned}b_d(x_n,x_m)&\le s[b_d(x_n,x_{n+1})+b_d(x_{n+1},x_m)]\\&\le sb_d(x_n,x_{n+1})+s^2b_d(x_{n+1},x_{n+2})+s^3b_d(x_{n+2},x_{n+3})+...\\&\le sk^nb_d(x_0,x_1)+s^2k^{n+1}b_d(x_0,x_1)+s^3k^{n+2}b_d(x_0,x_1)+...\\&=sk^nb_d(x_0,x_1)[1+sk+(sk)^2+(sk)^3+....]\\&\le\frac{sk^n}{1-sk}b_d(x_0,x_1).\end{aligned}$$

On taking limit for $n,m\to\infty$ we have $b_d(x_n,x_m)\to 0$ as $ks<1$. Therefore (x_n) is a b_d -Cauchy sequence in complete b-dislocated metric space (X,b_d). So there is some $u\in X$ such that (x_n) dislocated converges to u. Therefore the subsequences $\{Sx_{2n}\}\to u$ and $\{Tx_{2n+1}\}\to u$. Since $T,S:X\to X$ are continuous mappings we get: $Su=u$ and $Tu=u$. Thus, u is a common fixed point of T and S.

If consider that T is continuous and S not continuous we have that $Tu=u$. Using the contractive condition of theorem we have,

$$\begin{aligned}sd(Su,Tx_{2n+1})&\le cg[d(u,x_{2n+1}),d(u,Su),d(x_{2n+1},Tx_{2n+1}),\frac{d(u,Su)d(x_{2n+1},Tx_{2n+1})}{1+d(u,x_{2n+1})}]\\&\le cg[d(u,x_{2n+1}),d(u,Su),d(x_{2n+1},Tx_{2n+1}),\frac{d(u,Su)d(x_{2n+1},x_{2n+2})}{1+d(u,x_{2n+1})}].\end{aligned}$$

Taking in upper limit as $n\to\infty$, using lemma 2.10, property of g and result (4) we get

$$s\tfrac{1}{s}d(u,Su)\le cg[0,d(u,Su),0,0].$$

This inequality implies $d(u,Su)\le cd(u,Su)$ that means $d(u,Su)=0$. Thus $Su=u$ and u is a fixed point of S.

If consider (c) we have that, u is a common fixed point of S and T. Using the contractive condition of theorem, we obtain

$$\begin{aligned}sd(u,u)&=sd(Su,Tu)\\&\le cg[d(u,u),d(u,u),d(u,u),\frac{d(u,u)d(u,u)}{1+d(u,u)}]\\&=cd(u,u).\end{aligned}$$

The inequality above implies that $d(u,u)\le kd(u,u)$. So $d(u,u)=0$, since $0\le k=\frac{c}{s}<1$.

Uniqueness. Let suppose that u and v are two common fixed points of $T;S$. From condition (1) we have:

$$\begin{aligned} sd(u,v) &= sd(Su,Tv) \\ &\le cg[d(u,v), d(u,Su), d(v,Tv), \tfrac{d(u,Su)d(v,Tv)}{1+d(u,v)}] \qquad (5) \\ &= cg[d(u,v), d(u,u), d(v,v), \tfrac{d(u,u)d(v,v)}{1+d(u,v)}]. \end{aligned}$$

Replacing $v = u$ in (5) we get:

$$sd(u,u) \le cg[d(u,u), d(u,u), d(u,u), \tfrac{d(u,u)d(u,u)}{1+d(u,u)}] \le cd(u,u),$$

i.e. $d(u,u) \le \frac{c}{s} d(u,u) = kd(u,u)$. Since $0 \le k < 1$ we obtain $d(u,u) = 0$. Similarly replacing $u = v$ in (5), we obtain $d(v,v) = 0$. Again from (5) have $d(u,v) \le kd(u,v)$ since $0 \le k < 1$ get $d(u,v) = 0$, which implies $u = v$. Thus fixed point is unique.

Corollary 3.2. Let (X,d) be a complete b-dislocated metric space with parameter $s \ge 1$ and $T,S : X \to X$ two mappings satisfying the following contractive condition

$$sd(Sx,Ty) \le c\, g[d(x,y), d(x,Sx), d(y,Ty)]$$

for all $x, y \in X$ where $g \in G_3$ and $0 \le c < 1$. Then T and S have a unique common fixed point and if u is a common fixed point of S and T, then $d(u,u) = 0$.

Corollary 3.3. Let (X,d) be a complete dislocated metric space and $T,S : X \to X$ two mappings satisfying the following contractive condition

$$d(Sx,Ty) \le c\, g[d(x,y), d(x,Sx), d(y,Ty), \tfrac{d(x,Sx)d(y,Ty)}{1+d(x,y)}],$$

for all $x, y \in X$ where $g \in G_4$ and $0 \le c < 1$. Then T and S have a unique common fixed point and if u is a common fixed point of S and T, then $d(u,u) = 0$.

The following example supports our theorem.

Example 3.4. Let $X = [0,1]$ and $d(x,y) = x + y$, for all $x, y \in X$. It is clear that d is a dislocated metric on X. We define the self mappings $S,T : X \to X$ as follows

$$Sx = \begin{cases} \frac{1}{8}x, & x \in [0,1) \\ \frac{1}{6}, & x = 1 \end{cases} \quad \text{and} \quad Tx = \begin{cases} \frac{1}{5}x, & x \in [0,1) \\ \frac{1}{3}, & x = 1. \end{cases}$$

Note that S and T are discontinuous maps. Now we will show that the contractive condition of 3.3 is satisfied for constant $c \in [0,1)$ and taking the function $g(t_1,t_2,t_3,t_4) = \max\{t_1,t_2,t_3,t_4\}$. We have the following cases:

Case 1. Note that for all $x, y \in [0,1)$, we have

$$d(Sx,Ty) = d(\tfrac{x}{8}, \tfrac{y}{5}) = \tfrac{x}{8} + \tfrac{y}{5} \le \tfrac{1}{5}(x+y) = \tfrac{1}{5} d(x,y)$$

Case 2. Note that for $x = y = 1$, we have

$$d(Sx,Ty) = d(S1,T1) = d(\tfrac{1}{6},\tfrac{1}{3}) = \tfrac{1}{6} + \tfrac{1}{3} = \tfrac{1}{2} = \tfrac{1}{4}\cdot 2 = \tfrac{1}{4}d(x,y) .$$

Case 3. for $x \in [0,1)$ and $y = 1$, we have

$$d(Sx,Ty) = d(\tfrac{x}{8},\tfrac{1}{3}) = \tfrac{x}{8} + \tfrac{1}{3} \le \tfrac{1}{3}(x+1) = \tfrac{1}{3}d(x,y) .$$

Case 4. For all $y \in [0,1)$ and $x = 1$, we have

$$d(Sx,Ty) = d(\tfrac{1}{6},\tfrac{y}{5}) = \tfrac{1}{6} + \tfrac{y}{5} = \tfrac{5+6y}{30} \le \tfrac{1}{4}(1+y) = \tfrac{1}{4}d(x,y) .$$

Thus all conditions of corollary 3.3 are satisfied and $x = 0$ is a unique common fixed point of S and T .

Also we note that this theorem is not available in a usual metric space if $d(x,y) = |x-y|$ and in b-metric space $d(x,y) = |x-y|^2$ because if consider points $x = y = 1$ we will have

$$d(S1,T1) = |\tfrac{1}{6} - \tfrac{1}{3}| = \tfrac{1}{6} > cd(1,1) = 0$$

$$d(S1,T1) = |\tfrac{1}{6} - \tfrac{1}{3}|^2 = (\tfrac{1}{6})^2 = \tfrac{1}{36} > cd(1,1) = 0 .$$

So the contractive condition is failed in two cases.

Corollary 3.5. Let (X,d) be a complete dislocated metric space and $S: X \to X$ a self-mapping satisfying the following contractive condition

$$d(Sx,Sy) \le c\, g[d(x,y), d(x,Sx), d(y,Sy), \tfrac{d(x,Sx)d(y,Sy)}{1+d(x,y)}]$$

for all $x, y \in X$ where $g \in G_4$ and $0 \le c < 1$. Then, S has a unique fixed point and $d(u,u) = 0$

Example 3.6. Let $X = [0,10]$ and $d(x,y) = \frac{1}{2}(x+y)$, for all $x, y \in X$. It is clear that d is a dislocated metric on X and (X,d) is complete. Also d is not a metric on X . We define the self-mapping $S: X \to X$ by

$$Sx = \begin{cases} x-1, x \ne 0 \\ 0 \quad\quad , x = 0 \end{cases}$$

and take the function $g(t_1,t_2,t_3,t_4) = \max\{t_1,t_2,t_3,t_4\}$ and also choose the constant $c = \frac{9}{10}$. For $x, y \in \{0,1,....,10\}$, we have the following cases.

Case 1. For $x = y = 0$ have $d(Sx,Sy) = d(0,0) = 0$

Case 2. If $x = y > 0$, then

$$d(Sx,Sy) = d(x-1,x-1) = x-1 \le \tfrac{9}{10}x = \tfrac{9}{10}d(x,y) .$$

Case 3. If $x > y = 0$, then

$$d(Sx,Sy) = d(x-1,0) = \tfrac{1}{2}(x-1) \le \tfrac{9}{10}\tfrac{x}{2} = \tfrac{9}{10}d(x,y) .$$

Case 4. If $x > y > 0$, then

$$d(Sx,Sy) = d(x-1, y-1) = \tfrac{1}{2}(x+y-2) \le \tfrac{9}{10}\tfrac{1}{2}(x+y) = \tfrac{9}{10}d(x,y).$$

Thus all conditions of theorem are satisfied and S has a unique fixed point in X. Also we note that for $x = 1$ and $y = 10$ the contractive condition is failed in the usual metric.

Theorem 3.7. Let (X,d) be a complete b-dislocated metric space and $T,S : X \to X$ two self-mappings satisfying the condition:

$$sd(Sx,Ty) \le c\max\{d(x,y)+d(x,Sx), d(x,Sx)+d(y,Ty), d(x,y)+d(y,Ty), d(y,Ty)+\tfrac{d(x,Sx)d(y,Ty)}{1+d(x,y)}\}$$

for all $x, y \in X$ and $0 \le 2c < 1$. Then T and S have a unique common fixed point in X.

Proof. This theorem is corollary of theorem 3.1 if we use the function $g_2 \in G_4$.

Theorem 3.8. Let (X,d) be a complete b-dislocated metric space and $T,S : X \to X$ two self mappings satisfying the condition:

$$s^p d^p(Sx,Ty) \le c\max\{d^p(x,y), d^p(x,Sx), d^p(y,Ty), (\tfrac{d(x,Sx)d(y,Ty)}{1+d(x,y)})^p\},$$

for all $x, y \in X$ and $0 \le c < 1$.Then T and S have a unique common fixed point in X.

Proof. This theorem is taken as a corollary of theorem 1, if we use the function $g_4 \in G_4$.

Theorem 3.9. Let (X,d) be a complete b-dislocated metric space and $T,S : X \to X$ two self-mappings satisfying the condition:

$$s^2 d^2(Sx,Ty) \le c\max\{d(x,y)d(x,Sx), d(x,Sx)d(y,Ty), d(x,y)d(y,Ty), d(y,Ty)\tfrac{d(x,Sx)d(y,Ty)}{1+d(x,y)}\}$$

for all $x, y \in X$ and $0 \le 2c < 1$. Then T and S have a unique common fixed point in X.

Proof. This theorem is corollary of theorem 1, if we use the function $g_3 \in G_4$.

Remark 3.10. Results of the above theorems and corollaries are extended and unified of some classical fixed point results in metric spaces and generalization of results of the authors [1,2,9,10,18,19] and other results in dislocated metric spaces.

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