

EMBEDDINGS OF UNIVERSAL ALGEBRAS IN SEMIGROUPS

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A review of results concerning the problem of embeddings of universal algebras in semigroups is given in this paper.

1. Ω -subalgebras of semigroups. Let \underline{A} be an Ω -algebra and $\underline{S}=(S, \cdot)$ a semigroup such that $\underline{A} \subseteq \underline{S}$. We say that \underline{A} is an Ω -subalgebra of \underline{S} if there is a mapping $\omega \mapsto \bar{\omega}$ of Ω into S such that

$$*) \quad \omega_{\underline{A}}(a_1, \dots, a_n) = \bar{\omega} a_1 a_2 \dots a_n,$$

for all $\omega \in \Omega(n)$, $n \geq 0$, $a_1, \dots, a_n \in A$.

If \mathcal{C} is a class of semigroups then $\mathcal{C}(\Omega)$ denotes the class of Ω -algebras that are Ω -subalgebras of semigroups belonging to \mathcal{C} . We find it interesting looking for a convenient description of the class $\mathcal{C}(\Omega)$ when a corresponding description of the class \mathcal{C} is given. Thus, the well known Cohn-Rebane's Theorem, [1;p.185], [18;p.78] can be stated in the following way: $SEM(\Omega)$ is the class of all Ω -algebras. (By SEM we denote the class of semigroups.)

There are known convenient descriptions of the classes of all Ω -subalgebras of semigroups which belong to the following classes of semigroups:

ABSEM (abelian semigroups) ([23]),

CANSEM (cancellative semigroups) ([24]),

NILSEM (nilpotent semigroups) ([25]),

LZ (left zero semigroups),

*) $\omega_{\underline{A}}$ is the interpretation of the n -ary operator $\omega \in \Omega$; further on we write ω instead of $\omega_{\underline{A}}$; $\Omega(n)$ is the set of n -ary operators belonging to Ω .

RZ (right zero semigroups),

RCB (rectangular bands),

MIN (middle neutral semigroups, i.e. semigroups that satisfy the law $xyz=xz$),

SL (semilattices) ([11]),

ABPG (abelian periodical groups) ([12]),

$A_{r,m}$ (abelian semigroups that satisfy the law $x^r=x^{r+m}$, where $r \geq 0$, $m \geq 1$; $A_m=A_{0,m}$ is the class of abelian groups the orders of element of which are divisors of m) ([12]),

DISSEM (distributive semigroups, i.e. semigroups which satisfy both left and right distributive law) ([14]),

ABDISSEM (abelian distributive semigroups) ([13]),

LDISSEM (left distributive semigroups) ([15]),

RDISSEM (right distributive semigroups) ([15]).

We notice that there are not known convenient descriptions of $C(\Omega)$ for a lot of important classes C of semigroups (bands, inverse semigroups, periodic semigroups, abelian periodical semigroups, periodical groups,...).

There is only one result concerning the class of Ω -subalgebras of finite semigroups. Namely, it is shown in [22] that if $\Omega = \Omega(0) \cup \Omega(1)$ then every finite Ω -algebra is an Ω -subalgebra of a finite semigroup; in the same paper, it is also shown that if $\Omega \neq \Omega(0) \cup \Omega(1)$ then there exist finite Ω -algebras which are not Ω -subalgebras of finite semigroups.

2. Ω -varieties of semigroups. It follows by a more general result ([19;p.274], [16]) that if Q is a quasivariety of semigroups, then $Q(\Omega)$ is a quasivariety of Ω -algebras. Therefore, if V is a variety of semigroups, then $V(\Omega)$ is a quasivariety of Ω -algebras. We say that a variety V of semigroups is an Ω -variety if $V(\Omega)$ is a variety of Ω -algebras. Thus, if Ω is a given operator domain, then the set of varieties of semigroups is di-

vided into two disjoint subsets: Ω -varieties and varieties V of semigroups such that $V(\Omega)$ are proper quasivarieties. Clearly, if $\Omega = \Omega(0)$, then every variety of semigroups is an Ω -variety, as well; and, if $\Omega' = \Omega \setminus \Omega(0)$ then the set of Ω -varieties of semigroups coincides with the set of Ω' -varieties of semigroups. That is why we will assume that $\Omega \neq \Omega(0)$.

2.1. If $V \in \{\text{SEM}, \text{ABSEM}, \text{ABDISSEM}, \text{LZ}, \text{RZ}\} \cup \{A_{r,m} \mid m \geq 1\}$, then V is an Ω -variety for any operator domain Ω ([1], [7], [13], [23]).

2.2. If $V \in \{A_{r,m} \mid r \geq 2, m \geq 1\}$ or $V = \text{LDISSEM}$, then V is an Ω -variety iff $\Omega = \Omega(1)$ ([7], [15]).

2.3. If $m \geq 2$, then A_m is an Ω -variety if $\Omega \setminus \Omega(1) \neq \emptyset$ ([12]).

2.4. Let $V \in \{\text{DISSEM}, \text{RCB}, \text{MIN}\}$. V is an Ω -variety iff $\Omega = (\omega)$ consists of only one n -ary operator, with $n \geq 1$ ([14]).

2.5. RDISSEM is an Ω -variety iff $\Omega = \Omega(1)$ and $|\Omega| = 1$ ([15]).

We notice that in all mentioned examples of Ω -varieties V of semigroups, convenient axiom systems of the varieties $V(\Omega)$ are known. For example, if ξ, η are Ω -terms then $\xi = \eta$ is an identity equation in $\text{SL}(\Omega)$ iff ξ and η have the same content.

The situation is not the same if $V(\Omega)$ is a proper quasivariety. Namely, one proves that a semigroup variety V is not an Ω -variety by finding an algebra $\underline{A} = (A, \Omega)$ which satisfies all the identities of $V(\Omega)$, but $\underline{A} \notin V(\Omega)$. Consider, for example, the variety MIN . Clearly, an Ω -algebra $\underline{A} = (A, \Omega)$ satisfies all the identities of $\text{MIN}(\Omega)$ iff it satisfies the following identities:

$$\omega(x_1, \dots, x_{n-1}, z) = \omega(y_1, \dots, y_{n-1}, z), \quad (2.1)$$

$$\omega(x_1, \dots, x_{n-1}, \tau(y_1, \dots, y_{m-1}, z)) = \omega(x_1, \dots, x_{n-1}, z), \quad (2.2)$$

for any $\omega \in \Omega(n)$, $\tau \in \Omega(m)$.

Assume that $\omega \in \Omega(n)$, $\tau \in \Omega(m)$ are two different elements of Ω . Define an Ω -algebra \underline{A} with a carrier $A = \{a, b, c\}$ ($a \neq b \neq c \neq a$) as follows:

$$\rho(x_1, \dots, x_p) = \begin{cases} b & \text{if } x_p \in (a, b), \rho \in \Omega(p), \rho \neq \omega \\ a & \text{if } x_p \in (a, b), \rho = \omega \end{cases}$$

$$\rho(x_1, \dots, x_{p-1}, c) = c, \text{ for any } \rho \in \Omega(p).$$

It is clear that the obtained algebra \underline{A} satisfies all the identities (2.1) and (2.2). But $\underline{A} \notin \text{MIN}(\Omega)$, for if \underline{A} were an Ω -subalgebra of a semigroup $\underline{S} \in \text{MIN}$, then we would have:

$$\begin{aligned} a &= \omega(a^n) = \bar{\omega}a^n = \bar{\omega}c^n a = \omega(c^n)a \\ &= \tau(c^m)a = \bar{\tau}c^m a = \bar{\tau}a^m = \tau(a^m) \\ &= b. \end{aligned}$$

This shows that $\text{MIN}(\Omega)$ is a proper quasivariety.

It can be easily seen that every algebra belonging to $\text{MIN}(\Omega)$ satisfies any quasiidentity of the following form:

$$\omega(x_1, \dots, x_n) = \tau(y_1, \dots, y_m) \Rightarrow \omega(z_1, \dots, z_{n-1}, z) = \tau(z_1, \dots, z_{m-1}, z), \quad (2.3)$$

for every $\omega \in \Omega(n)$, $\tau \in \Omega(m)$. The converse is also true, i.e. if an Ω -algebra \underline{A} satisfies all the formulas (2.1), (2.2) and (2.3), then $\underline{A} \in \text{MIN}(\Omega)$. Let us give a sketch of the proof of the last proposition.

Assume, namely, that (A, Ω) satisfies (2.1), (2.2) and (2.3), and let $B = A \cup \Omega$, $A \cap \Omega = \emptyset$. Define an operation \cdot on the set $F = B \times B \cup B$ in the following way:

$$\begin{aligned} b_1 \cdot b_2 &= (b_1, b_2), \quad b_1 \cdot (b_2, b_3) = (b_1, b_3) \\ (b_1, b_2) \cdot b_3 &= (b_1, b_3), \quad (b_1, b_2) \cdot (b_3, b_4) = (b_1, b_4), \end{aligned}$$

$b_i \in B$. In such a way we get a semigroup $\underline{F} \in \text{MIN}$. Let \approx be the minimal congruence on \underline{F} such that:

$$a = \omega(a_1, \dots, a_n) \text{ in } \underline{A} \Rightarrow a \approx (\omega, a_n) \text{ in } \underline{F}.$$

Then, it can be shown that:

$$a, b \in A \Rightarrow (a \approx b \Rightarrow a = b),$$

and this implies that \underline{A} can be embedded as an Ω -subalgebra into the semigroup $\underline{S} = \underline{F}/\approx$. Thus, $\underline{A} \in \text{MIN}(\Omega)$.

Clearly, if $\Omega = \{\omega\}$ consists of only one element, then (2.3) is always satisfied, and therefore in this case $\text{MIN}(\Omega) = \text{MIN}(\omega)$ is a variety.

We point out that $\text{MIN}(\Omega) = \text{RCB}(\Omega)$, for every operator domain Ω .

We notice that in all other known examples of semigroup varieties \mathcal{V} such that $\mathcal{V}(\Omega)$ is a proper quasivariety (and $\mathcal{V} \neq \mathcal{A}_m$) we do not know a convenient axiom system of the quasivariety $\mathcal{V}(\Omega)$.

3. Semigroups of finitary operations. Let A be a nonempty set, n be a positive integer, and $O_n(A)$ be the set of all n -ary operations on A . Then $O(A) = \bigcup_{n \geq 1} O_n(A)$ is the set of all non-nullary operations on A . If a binary operation \bullet is defined on $O(A)$ in the usual way, i.e. by:

$$(\omega \bullet \tau)(x_1, \dots, x_{m+n-1}) = \omega(\tau(x_1, \dots, x_m), \dots, x_{m+n-1}),$$

where $\omega \in O_n(A)$, $\tau \in O_m(A)$, then we get a semigroup $(O(A), \bullet)$. Any subsemigroup Γ of this semigroup is said to be a semigroup of operations on A , and then we have a Γ -algebra $\underline{A} = (A, \Gamma)$. By Cohn-Rebane's Theorem if Γ is a semigroup of operations on A , then the corresponding Γ -algebra (A, Γ) is a Γ -subalgebra of a semigroup. And, if \mathcal{C} is a class of semigroups, then we denote by $\mathcal{C}^{(1)}$ the class of semigroups of operations Γ such that the corresponding Γ -algebras are Γ -subalgebras of semigroups belonging to \mathcal{C} . If, moreover, Γ is a semigroup of operations on A such that the corresponding Γ -algebra $\underline{A} = (A, \Gamma)$ can be embedded as a Γ -subalgebra in a semigroup $\underline{S} \in \mathcal{C}$ in such a way that the mapping $\omega \mapsto \bar{\omega}$ is a homomorphism (an injective homomorphism), then we say that Γ belongs to the class $\mathcal{C}^{(2)}$ ($\mathcal{C}^{(3)}$).

It is clear that:

3.1. $\mathcal{C}^{(3)} \subseteq \mathcal{C}^{(2)} \subseteq \mathcal{C}^{(1)}$, for any class \mathcal{C} of semigroups.

We do not know any class of semigroups \mathcal{C} , such that $\mathcal{C}^{(2)}$ is a proper subclass of $\mathcal{C}^{(1)}$. Below we state some known results.

3.2. If $\mathcal{C} \in \{\text{SEM}, \text{ABSEM}, \text{CANSEM}, \text{ABCANSEM}, \text{ABGP}\}$, then, $\mathcal{C}^{(1)} = \mathcal{C}^{(2)} = \mathcal{C}^{(3)}$ ([8]).

3.3. If $\mathcal{C} \in \{\text{NILSEM}, \text{ABNILSEM}, \text{MIN}, \text{RCB}\} \cup \{A_m \mid m \geq 2\}$ then $\mathcal{C}^{(3)} \subset \mathcal{C}^{(2)} = \mathcal{C}^{(1)}$, and the inclusion is strict ([8]).

4. n-subsemigroups of semigroups. Let $S = (S, \cdot)$ be a semigroup and Q a subset of S such that:

$$x_1, \dots, x_n \in Q \Rightarrow x_1 \cdot x_2 \cdot \dots \cdot x_n \in Q,$$

where $n \geq 2$. Then we say that Q is an n -subsemigroup of S . If we define an n -ary operation $[\]$ on Q by:

$$[x_1, \dots, x_n] = x_1 \cdot \dots \cdot x_n,$$

then we get that $[\]$ is an associative operation, i.e. $(Q, [\])$ is an n -semigroup.

If \mathcal{C} is a class of semigroups, then we denote by $\mathcal{C}(n)$ the class of n -semigroups which are n -subsemigroups of semigroups belonging to \mathcal{C} . A variety of semigroups \mathcal{V} is said to be an n -variety of semigroups iff $\mathcal{V}(n)$ is a variety of n -semigroups.

4.1. Each of the varieties of semigroups $\text{SEM}, \text{ABSEM}, \text{LZ}, \text{RZ}, \text{DISSEM}, \text{RCB}, \text{MIN}, A_{r,m}$ is an n -variety for any $n \geq 2$ ([4], [5], [20], [21]).

4.2. Let $r \geq 0$ and $m \geq 1$. The variety $P_{r,m}$ of semigroups satisfying the law $x^r = x^{r+m}$ is an n -variety iff $r \in \{0, 1\}$ or $n-1$ is a divisor of m ([4]).

4.3. All members of the set of varieties:

$$\{V \mid \text{ABSEM} \subseteq V\} \cup \{V \mid V \subseteq P_{1,m}\}$$

are n -varieties.

4.4. $\text{LDISSEM}(n)$ and $\text{RDISSEM}(n)$ are proper quasivarieties, for any $n \geq 3$ ([20]).

4.5. Let $n \geq 3$. There exist infinite many varieties of abelian semigroups which are not n -varieties ([6]).

The results stated in this section and in the section 2 suggests the conjecture that if a variety of semigroups is an ω -variety, where ω is an n -ary operator, then it is an n -variety as well.

4.6. An n -semigroup $(Q, [\])$ belongs to the quasivariety $CAN(n)$ iff it is cancellative. Every cancellative n -semigroup $(Q, [\])$ is an n -subsemigroup of a universal cancellative semigroup $(Q^-, \cdot) = \underline{Q}^-$, and then:

$$(Q, [\]) \in GP(n) \text{ iff } \underline{Q}^- \in GP(2) \quad ([17]).$$

The last result can be used for obtaining an axiom system of the quasivariety $GP(n)$. Namely, we can take some of the known axiom system of $GP(2)$ (see, for example, [1;VII.3]) and then we can transform it into an axiom system of $GP(n)$.

5. Ω -subsemigroups of semigroups. The class of Ω -subsemigroups of semigroups is an extension of the class of n -subsemigroups of semigroups. Assume first that $\Omega(0) = \emptyset$. An Ω -algebra $\underline{A} = (A, \Omega)$ is an Ω -subsemigroup of a semigroup $\underline{S} = (S, \cdot)$ if $A \subseteq S$ and

$$\omega(a_1, \dots, a_n) = a_1 \dots a_n,$$

for every $\omega \in \Omega(n) \subseteq \Omega$ and $a_1, \dots, a_n \in A$. If \mathcal{C} is a class of semigroups, then we denote by $\mathcal{C}[\Omega]$ the class of Ω -subsemigroups of semigroups belonging to \mathcal{C} .

Clearly, if $\omega \in \Omega(1)$ and if $\underline{A} = (A, \Omega)$ is an Ω -subsemigroup of a semigroup, then we have $\omega(a) = a$, for every $a \in A$, and that is why below we assume that $\Omega(1) = \emptyset$ and $\Omega \neq \emptyset$.

A convenient description of the quasivariety $SEM[\Omega]$ is given in [2]. Almost all known results concerning Ω -subsemigroups of semigroups could be found in [5], and we will state only a few of them.

5.1. Let $J_\Omega = \{n-1 \mid \Omega(n) \neq \emptyset\}$, and let d_Ω be the greatest common divisor of numbers belonging to J_Ω . Then, $SEM[\Omega]$ is a variety iff $d_\Omega \in J_\Omega$.

5.2. If $\forall e \{LZ, RZ, MIN, RCB, SL\}$, then $V[\Omega]$ is a variety.

There are also known corresponding description of $CAN[\Omega]$, $GP[\Omega]$, and of some subclasses of $SEM[\Omega]$.

6. Generalized subalgebras of semigroups. The classes of Ω -subalgebras of semigroups and Ω -subsemigroups of semigroups are special kinds of Δ -subalgebras of semigroups defined in the following way.

Let C be a set and $C_0 = C \cup \{e\}$, where $e \notin C$. Assume now that Ω is an operator domain and $\Delta: \omega \mapsto \omega^\Delta$ a mapping from Ω into the set $\bigcup_{k \geq 1} C_0^k$ such that:

$$(i) \omega \in \Omega(n) \Rightarrow \omega^\Delta = (\omega_0, \omega_1, \dots, \omega_n) \in C_0^{n+1}$$

(ii) if $c \in C$ then there exists an $\omega \in \Omega(n)$ and an $i \in \{0, \dots, n\}$ such that $c = \omega_i$.

An Ω -algebra $\underline{A} = (A, \Omega)$ is said to be a Δ -subalgebra of a semigroup $\underline{S} = (S, \cdot)$ if $A \subseteq S$ and there is a mapping $d \mapsto \bar{d}$ from C_0 into S^1 *) such that:

$$(iii_1) \bar{e} = 1, \quad (iii_2) c \in C_0 \Rightarrow \bar{c} \in S, \text{ and}$$

$$(iv) \omega(a_1, \dots, a_n) = \bar{\omega}_0 a_1 \bar{\omega}_1 a_2 \dots \bar{\omega}_{n-1} a_n \bar{\omega}_n$$

for any $\omega \in \Omega(n)$, $n \geq 0$, $a_i \in A$.

We say that Δ is a semigroup representation of Ω .

If \mathcal{C} is a class of semigroups, then we denote by $\mathcal{C}(\Delta)$ the class of Δ -subalgebras of semigroups belonging to \mathcal{C} .

6.1. If $C = \Omega$, and $\Omega^\Delta = (\omega, e, \dots, e)$ for any $\omega \in \Omega$, then $\mathcal{C}(\Delta) = \mathcal{C}(\Omega)$.

6.2. If $C = \emptyset$, then $\mathcal{C}(\Delta) = \mathcal{C}[\Omega]$.

6.3. Let $\Omega(0) \cup \Omega(1) = \emptyset$, $C = \Omega$, and $\Omega^\Delta = (e, \omega, e, \dots, e)$ for any $\omega \in \Omega$. Then $\text{SEM}(\Delta)$ is a variety of Ω -algebras ([3]).

6.4. Let $\{\hat{\omega} \mid \omega \in \Omega\}$ be a collection of disjoint sets such that:

$$\omega \in \Omega(n) \Rightarrow \hat{\omega} = \{\omega_0, \omega_1, \dots, \omega_n\}, \omega_i \neq \omega_j \text{ if } i \neq j,$$

and let $C = \bigcup_{\omega \in \Omega} \hat{\omega}$. Define a semigroup representation Δ by:

$\omega^\Delta = (\omega_0, \omega_1, \dots, \omega_n)$. Then $\text{SEM}(\Delta)$ is the class of all Ω -algebras ([10]).

*) $S^1 = S$ if \underline{S} has a unity, and if this is not the case then $1 \notin S$ and 1 is the unity of \underline{S}^1 .

It follows from 6.2 and 5.1 that there exists a semigroup representation Δ such that $\text{SEM}(\Delta)$ is not a variety. This result suggests the question of giving a description of semigroup representations Δ such that $\text{SEM}(\Delta)$ is a variety.

Before giving an answer to this question we have to define a mapping $\xi \mapsto \xi^\Delta$ of Ω -terms into $C \cup \{\cdot\}$ -terms which is induced by a semigroup representation Δ . This mapping is defined, namely, in the following way.

(v) If x is a variable, then $x^\Delta = x$.

(vi) If $\omega \in \Omega(n)$ and ξ_1, \dots, ξ_n are Ω -terms, then

$$(\omega(\xi_1, \dots, \xi_n))^\Delta = \omega_0 \xi_1^{\Delta \omega_1} \xi_2^{\Delta \omega_2} \dots \xi_{n-1}^{\Delta \omega_{n-1}} \xi_n^{\Delta \omega_n}.$$

(Here, the elements of C are interpreted as constants and e as an "empty" symbol.)

If t is a $C \cup \{\cdot\}$ -term such that $t = \xi^\Delta$ for an Ω -term ξ , then we say that t is a Δ -term.

Now we can give a description of semigroup representations Δ such that the classes $\text{SEM}(\Delta)$ are varieties.

6.5. Let $\Omega(0) = \emptyset$. $\text{SEM}(\Delta)$ is a variety iff the following condition is satisfied:

⊗ If t', t'' and t_1, t_2 are Δ -terms, then $t_1 t_2$ is a Δ -term as well.

(In the paper [26] it is shown that ⊗ is a sufficient condition, and the necessity of this condition is shown in [21].)

Let Δ be a semigroup representation of Ω , and $\underline{A} = (A, \Omega)$ an Ω -algebra. Then the semigroup \underline{A}^Δ with the presentation:

$$\langle A \cup C; \{a = \omega_0 a_1 \dots \omega_{n-1} a_n \omega_n \mid a = \omega(a_1, \dots, a_n) \text{ in } \underline{A}\} \rangle \quad (6.1)$$

is called a universal Δ -enveloping semigroup of \underline{A} . If \mathcal{V} is a variety of semigroups, then by $\underline{A}^\Delta_{\mathcal{V}}$ is denoted the semigroup with the presentation (6.1) in the variety \mathcal{V} .

Now we can give the following characterization of $\mathcal{V}(\Delta)$.

6.6. $\underline{A} \in \mathcal{V}(\Delta)$ iff:

$$a, b \in A \Rightarrow (a=b \text{ in } \underline{A}_{\mathcal{V}}^{\Delta} \Rightarrow a=b \text{ in } \underline{A}) \quad (6.2)$$

We notice that the statement that $\mathcal{V}(\Delta)$ is a variety one proves by showing that if an Ω -algebra \underline{A} satisfies all the identities of $\mathcal{V}(\Delta)$, then it satisfies (6.2), as well.

6.7. If $\langle B; \Lambda \rangle$ is a presentation of \underline{A} in the class of Ω -algebras, then $\underline{A}_{\mathcal{V}}^{\Delta}$ has a presentation $\langle B \cup C; \Lambda^{\Delta} \rangle$ in \mathcal{V} . Moreover, if Δ is a semigroup representation of the kind given in 6.1, 6.3 or 6.4, then $\langle B; \Lambda \rangle$ is solvable iff $\langle B \cup C; \Lambda^{\Delta} \rangle$ is solvable ([10]).

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СМЕСТУВАЊЕ НА УНИВЕРЗАЛНИ АЛГЕБРИ ВО ПОЛУГРУПИ

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Резиме

Во оваа работа се дава приказ на резултати што се однесуваат за сместување на универзални алгебри во полугруппе.