n-SEMIGROUPS WITH IDEMPOTENTS IN WHICH ALL n-SUBSEMIGROUPS ARE LEFT IDEALS

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Introduction. An n-semigroup S is said to be λ-n-semigroup if and only of all n-subsemigroups of S are left ideals of S. In [3] we have established some general properties of λ-n-semigroups. In this article we shall give a description of the structure of $\overline{\lambda}$-n-semigroups, i.e. λ-n-semigroups with idempotents.

1. The structure of unipotent $\overline{\lambda}$-n-semigroups. Let S be a unipotent $\overline{\lambda}$-n-semigroup with e as its idempotent and let $[\vartheta]$ denotes the operation in S. If we put $A=\{x\in S|x^{n+1}\neq e\}$, $B=\{x\in S\,|\,x\neq e,\ x=y^{n+1},\ y\in S\}$, $T=\{x\in S|x\neq e,\ x^{n+1}=e,\ x=y^{n+1}$ for no $y\in S\}$ and $R=T\cup\{e\}$, then S will be equal to the union of A, B and R. So, taking into account Theorem 2, (i), of [3], we have the following.

Lemma 1. $S=A\cup B\cup R$ where the union is disjoint and R is a left ideal in S. ∎

As in [3], we call R the reduced unipotent $\overline{\lambda}$-n-subsemigroup of S.

If we put $g(x)=x^{n+1}$, $x\in A$, then by the definitions of A and B given above we obtain that.

(1) $$g\colon A\to B \text{ is a surjection.}$$

Let $Q=A\cup T$ and let us put $f(x_0,\ldots,x_n)=1$ when $[x_0x_1\cdots x_n]\neq e$ and $f(x_0,x_1,\ldots,x_n)=0$ when $[x_0x_1\cdots x_n]=e$, $x_j\in Q$, $j=0,1,\ldots,n-1$, $x_n\in A$. Then f will be a well-defined mapping from $Q^n\times A$ to $\{0,1\}$ such that

(2) $$f(a,a,\ldots,a)=1,\quad a\in A.$$

Let f be defined on $(x_0,x_1,\ldots,x_n)$ and let for some x_j, $x_j=[y_1\cdots\cdots y_nx_j]$ where $y_i\in R\setminus\{e\}$, $i=1,2,\ldots,n$; by Lemma 7 of [3] $x_j\in R\setminus\{e\}$, so $j\neq n$ since $x_n\in A$. For the same reason, there is no product of the form $[\cdots x_n]$ which is equal to x_n. So, by Lemma 5 of [3], there are two possibilities for $p=[y_{n-j+1}\cdots x_j\cdots x_n]$: $p=e$ or $x_n{}^{n+1}$. If $p=e$, then $[x_0x_1\cdots x_n]=[x_0\cdots x_{j-1}y_1\cdots y_{n-j}\,p]=e$ ([3] (1) and so $f(x_0, x_1,\ldots, x_n)=0$. If $p=x_n{}^{n+1}$, then $[x_0x_1\cdots x_n]=[[x_0\cdots x_{j-1}y_1\cdots y_{n-j}\,x_n\cdots x_n]=[e\,x_n\cdots x_n]$ or $[(x_n{}^{n+1})\,x_n\cdots x_n]$ and in both cases $[x_0x_1\cdots x_n]=e$ ([3], Theorem 2 (iii)). This proves the following property of f:

(3) $f(x_0, x_1,\ldots, x_n)=0$ if f is defined on $(x_0, x_1,\ldots, x_n)$ and if $x_j=[y_1\cdots y_n\,x_j]$ for some $j=0, 1,\ldots, n-1$, where $y_i\in R\setminus\{e\}$, $i=1, 2,\ldots, n$.

From the definitions of f and g it follows that $[x_0x_1\cdots x_n]=g(x_n)$ if $f(x_0, x_1,\ldots, x_n)=1$ and $[x_0x_1\cdots x_n]=e$ if $f(x_0, x_1,\ldots, x_n)=0$. Let us suppose that f is not defined on $(x_0, x_1,\ldots, x_n)$ and that at least one x_j is not in R. If $x_k=e$ for some $k=0, 1,\ldots, n$, then by Theorem 2 of [3] it follows that $[x_0x_1\cdots x_n]=e$. If all x_j are different from e, then we have the following two subcases to consider: (i) $x_n\notin A$, or (ii) $x_i\in B$ for some $i<n$. Since not all x_j are in R, $[x_0x_1\cdots x_n]\neq x_n$ and if (i) is true, the $x_n{}^{n+1}=e$ which implies that $[x_0x_1\cdots x_n]=e$. If (ii) is true, then $x_i=y^{n+1}$ where $y\notin R$ since on the contrary we would have that $y^{n+1}=e=x_i$ which contradicts the choice of x_i as an element of B. Now,

$$[x_0x_1\cdots x_n]=[x_0\cdots x_{i-1}\,y\cdots y\,[y\cdots y x_{i+1}\cdots x_n]].$$

Since $y\notin R$, $[y\cdots y\,x_{i+1}\cdots x_n]\neq x_n$; if $[y\cdots y\,x_{i+1}\cdots x_n]=e$ then $[x_0x_1\cdots x_n]=e$, and if $[y\cdots y\,x_{i+1}\cdots x_n]=x_n{}^{n+1}$ then

$$[x_0x_1\cdots x_n]=[[x_0\cdots x_{i-1}\,y\cdots yx_n]\,x_n\cdots x_n].$$

Again, $y\notin R$ implies $[x_0\cdots x_{i-1}\,y\cdots y\,x_n]\neq x_n$ and in both cases, $[x_0\cdots x_{i-1}\,y\cdots y\,x_n]=e$ or $x_n{}^{n+1}$ it follows that $[x_0x_1\cdots x_n]=e$. In summary we obtain,

(4) $[x_0x_1\cdots x_n]=g(x_n)$, if $f(x_0, x_1,\ldots, x_n)=1$, $[x_0x_1\cdots x_n]=e$ in all other cases when $x_j\notin R\setminus\{e\}$ for at least one $0\leqslant j\leqslant n$.

Now, let A, B and R be pairwise disjoint sets and let R be a reduced unipotent $\overline{\lambda}$-n-semigroup with e as its idempotent. Let $f\colon Q^n\times A\to\{0, 1\}$ and $g\colon A\to B$ be two mappings such that (1) — (3) hold, where $Q=A\cup(R\setminus\{e\})$ and $[\cdots]$ denotes the operation in R. If we define in $S=A\cup B\cup R$ an $(n+1)$-ary operation $(\cdots)$ by

$$(x_0x_1\cdots x_n)=\begin{cases} g(x_n) \text{ if } f(x_0\,x_1\ldots, x_n)=1\\ [x_0x_1\cdots x_n] \text{ if all } x_j\in R\\ e \text{ in all other cases,}\end{cases}$$

then $S((\cdots))$ becomes a unipotent $\overline{\lambda}$-n-semigroup. The proof of this statement follows from the following considerations.

Let $x_j \in S$, $j = 0, 1, \ldots, 2n$. If all $x_j \in R$, then the associativity of $(\cdots)$ follows from the associativity of $[\cdots]$. Let, for example, $x_s \notin R$ and let us consider the „product"

$$p = (x_0 \cdots x_{i-1} (x_i \cdots x_{i+n}) x_{i+n+1} \cdots x_{2n})$$

where $i = 0, 1, \ldots, n$ is arbitrary chosen. If $i \leqslant s \leqslant i+n$, then $(x_i \cdots \cdots x_{i+n}) = g(_{i+n}) \in B$ or $(x_i \cdots x_{i+n}) = e$, and in both cases f is not defined on $(x_0, \ldots, x_{i-1}, (x_i \ldots x_{i+n}), x_{i+n+1}, \ldots, x_{2n})$, so $p = e$. Let $i > s$ or $i + n < s$ and let us put $(x_i \cdots x_{i+n}) = y$. According to the previous, we have to consider only the case $y = x_{i+n} \neq e$; we take that all x_{i+k}, $k = 0, 1, \ldots, n$ belong to $R \setminus \{e\}$, since on the contrary we will have the case already considered. But now, by (3), we have that $f(x_0, \ldots, x_{i-1}, y, x_{i+n+1}, \ldots, x_{2n}) = 0$ and again $p = e$, This proves that $S((\cdots))$ is an n-semigroup.

The idempotent of R is an idempotent in S, too. If $x \in S$ is an idempotent, and if $x \in R$, then $x = e$ since R is unipotent; if $x \notin R$, then $x = (x \cdots x)\, x \cdots x = e$, as we saw above, and S is unipotent.

Let U be an n-subsemigroup of S and let $x_j \in S$, $j = 1, 2, \ldots, n$ $y \in U$. Then $z = (x_1 \cdots x_n\, y) = e$, y or $g(y)$. In the first two cases it is obvious that $z \in \langle y \rangle \subseteq U$. In the last one $y \in A$, so that $f(y, y, \ldots, y) = 1$, and, by the definition of $(\cdots)$, it follows that $g(y) = (yy \cdots y)$ and again $z \in \langle y \rangle$. So, $(S \cdots SU) \subseteq U$ which means that S is a $\overline{\lambda}$-n-semigroup. We shall call the n-semigroup $S((\cdots))$ an (A, B, R, f, g) n-semigroup, $S = (A, B, R, f, g)$.

If we summarize the previous discussion, we obtain the following

Theorem 1. An n-semigroup S is a unipotent $\overline{\lambda}$-n-semigroup if and only if $A = (S, B, R, f, g)$. ∎

Let us prove, now, the following

Theorem 2. Let $h: S \to S'$ be a mapping, where $S = (A, B, R, f, g)$ and $S' = (A', B', R', f', g')$ are uuipotent $\overline{\lambda}$-n-semigrous. Then h is a homomorphism if and only if:

(1) $h_{|R}$ is a homomorphism from R to S';

(2) $h(R) \subseteq R' \cup B'$;

(3) $h(B) \subseteq B' \cup \{e'\}$;

(4) $h(g(x)) = g'(h(x))$ for all $x \in h^{-1}(A')$

(5) $f(x_0, x_1 \ldots, x_n) = f'(h(x_0), h(x_1), \ldots, h(x_n))$ if $x_n \in h^{-1}(A)$ and $x_i \in h^{-1}(A' \cup (R' \setminus \{e'\}))$, $i = 0, 1, \ldots, n-1$;

(6) $f(x_0, x_1, \ldots, x_n) = 0$ if $x_n \in h^{-1}(A')$, $x_j \in A \cup (R \setminus \{e\})$ for every $j = 0, 1, \ldots, n-1$ and $x_k \notin h^{-1}(A' \cup (R' \setminus \{e'\}))$ for some $k = 0, 1, \ldots, n-1$;

(7) if $h(x) \notin A'$, then $h(x^{n+1}) = e'$.

Proof. Let h be a homomorphism. (1) is obviously true. If $h(e) = u$, then $u = h(e^{n+1}) = [h(e)]^{n+1} = u^{n+1}$, so $u \in S'$ is an idempotent. Since S' is unipotent, $u = e'$. Let $z \in R$ and let $h(z) = v$; then $z^{n+1} = e$ implies $v^{n+1} = e'$, so $v \notin A'$ which proves (2). If $x \in B$, $x = y^{n+1}$, then $h(x) = [h(y)]^{n+1} \subseteq \subseteq B' \cup \{e'\}$, so (3) is true. From (2) and (3) it follows that $h^{-1}(A') \subseteq A$. Now, it is easily seen that (4) is satisfied. By the presumptions in (5), f' is defined on $(h(x_0), \ldots, h(x_n))$. We have seen that $h^{-1}(A') \subseteq A$; similarly, taking into account that $h(e) = e'$, we have that $h^{-1}(T') \subseteq A \cup T$, where $T = R \setminus \{e\}$, $T' = R' \setminus \{e'\}$. This implies that f is defined on $(x_0, \ldots, x_n)$. If $f(x_0, \ldots, x_n) = 0$, then $[x_0 \cdots x_n] = e$ which implies that $[h(x_0), \ldots, h(x_n)] = e'$ and therefore, $f'(h(x_0), \ldots, h(x_n)) = 0$. Now, let $f(x_0, \ldots, x_n) = 1$; then $[x_0 \cdots x_n] = g(x_n)$ and according to (4), we have that $[h(x_0) \cdots h(x_n)] = g'(h(x_n))$, i.e. $f'(h(x_0), \ldots, h(x_n)) = 1$ which proves (5). If for some k, x_k does not belong to $h^{-1}(A' \cup T')$, then f' will not be defined on $(h(x_0), \ldots, h(x_n))$, so $[h(x_0) \cdots h(x_n)] = e'$. From the other presumptions in (6) it follows that f is defined on $(x_0, \ldots, x_n)$. If $f(x_0, \ldots, x_n) = 1$ then by (4) we would have that $[h(x_0) \cdots h(x_n)] = g'(h(x_n)) \in B'$ which is impossible, and so it must be $f(x_0, \ldots, x_n) = 0$ which means that (6) is true. Finally, if $h(x) \notin A'$, then f' will not be defined on $(h(x), \ldots, h(x))$, so, $[h(x) \cdots \cdots h(x)] = e'$, i.e. $h(x^{n+1}) = e'$.

Conversely, let for the mapping $h: S \to S'$ be satisfied the statements (1)—(7). Since S' is unipotent, from (1) it follows that $h(e) = e'$. So, taking into account this and (1), all we have to prove is that $h([x_0 \cdots x_n]) = [h(x_0) \cdots h(x_n)]$ when $x_j \neq e$ for all $j = 0, 1, \ldots, n$ and $x_k \notin R$ for some $k = 0, 1, \ldots, n$. Let us put $u = h([x_0 \cdots x_n])$, $v = [h(x_0) \cdots h(x_n)]$, $X = (x_0, \ldots, x_n)$ and $h(X) = (h(x_0), \ldots, h(x_n))$. If $x_n \notin A$, then f is not defined on X and so, $u = e'$; but $x_n \notin A$ implies, by (2) and (3), that $h(x_n) \notin A'$ which implies that f' is not defined on $h(X)$ and so, $x = e'$. Let $x_n \in A$; we shall consider separately the following two cases: $[x_0 \cdots \cdots x_n] = e$ and $[x_0 \cdots x_n] = g(x_n)$.

Let $[x_0 \cdots x_n] = e$; we have the following two subcases to consider: (a) f is not defined on X, (b) $f(x_0, \ldots, x_n) = 0$. Let (a) be true. If $h(x_n) \notin A'$, then f' is not defined on $h(X)$, so that $v = e' = h(e) = u$. If $h(x_n) \in A'$, then, since f is not defined on X, there must be some $x_i \in B$ and, according to (3), $h(x_i) \notin A' \cup T'$ which means that f' is not defined on $h(X)$, and again we have that $u = v$. Now, let (b) be true. Let us suppose that $x_n \in h^{-1}(A')$ and $x_j \in h^{-1}(A' \cup T')$ for all $j = 0, 1, \ldots, n-1$, since on the contrary, as in (a), f' will not be defined on $h(X)$ and we shall come to the needed conclusion. Now, by (5), we have that $f'(h(X)) = 0 = f(X)$, so that $v = e' = u$.

Let, finally, $[x_0 \cdots x_n] = g(x_n) = x_n{}^{n+1}$; then $f(X) = 1$. If $x_n \notin h^{-1}(A)$, then $f'(h(X)) = 0$, so that, by (7), $v = e' = h(x_n{}^{n+1}) = u$. If $x_n \in h^{-1}(A')$, then by (6) $x_j \in h^{-1}(A' \cup T')$ for all $j=0, 1, \ldots, n-1$ and, by (5), we have that $f'(h(x_n)) = 1$. Now, by (4), it follows that $v = g'(h(x_n)) = h(g(x_n)) = u$. ∎

Theorem 3. Let $S = (A, B, R, f, g)$ and $S' = (A', B', R', f', g')$ be two $\overline{\lambda\text{-}n}$-semigrous, and $\xi: R \to R'$, $\eta: A \to A'$, $\zeta: B \to B'$ three bijections. Let $h: S \to S'$ be an extension of ξ, η, ζ. Then h is an isomorphism if and only if the following statements are satisfied:

(1) ξ is an isomorphism,

(2) $hg = g'h$

(3) if $(x_0, \ldots, x_n) \in Q^n \times A$, then $f(x_0, \ldots, x_n) = f'(h(x_0), \ldots, h(x_n))$ where $Q = A \cup (R \setminus \{e\})$

and every isomorphism from S into S' is obtained in that way.

Proof. If h is an isomorphism, then by Theorem 2, (1), (2) and (3) of this Theorem are satisfied. Conversely, if $h: S \to S'$ is an extension of ξ, η and ζ such that (1), (2) and (3) are satisfied, then the statements (1) — (7) of Theorem 2 are satisfied; the statement (6) of Theorem 2 is vacuously true, since the presumptions in that statement can not be realized. Now, by Theorem 2, h is a homomorphism and as a bijection, h is an isomorphism.

Let $\varphi: S \to S'$ be an isomorphism. In the proof of Theorem 2 we have seen that $\varphi^{-1}(A') \subseteq A$ and by symetry, since $\varphi^{-1}: S' \to S$ is an isomorphism, it follows that $\varphi(A) \subseteq A'$; from $\varphi^{-1}(A') \subseteq A$ it follows that $A' \subseteq \varphi(A)$ which together with $\varphi(A) \subseteq A'$ imply that $\varphi(A) = A'$. So, if we put $\eta = \varphi_{|A}$ then η becomes a bijection if we consider η as a mapping from A into A'. It can be shown, similarly, that $\varphi(e) = e'$ and (3) of Theorem 2 imply $\varphi(B) = B'$ and then that, from $\varphi(B) = B'$ and (2) of Theorem 2 it follows that $\varphi(R) = R'$. In the same way as above, $\zeta = \varphi_{|B}$ and $\xi = \varphi_{|R}$ can be considered se bijections, and φ will be an extension of ξ, η, ζ. ∎

The structure of arbitrary $\overline{\lambda\text{-}n}$-semigroups. Let S be a $\overline{\lambda\text{-}n}$-semigroup. To describe the structure of S, according to Theorem 2 of [3], we must see how the product $[x_0 x_1 \cdots x_n]$ is determined, when not all x_j belong to the same unipotent n-subsemigroup $S(e)$.

Let J be a set equivalent to the set E of all the idempotents of S and let us take J as an index set for E, i.e. $E = \{e_j \in S \mid e_j^{n+1} = e_j,\ j \in J\}$. If $S(e_j) = (R_j, A_j, B_j, f_j, g_j)$ is the unipotent $\overline{\lambda\text{-}n}$-subsemigroup of S which corresponds to the idempotent e_j, then $R = \bigcup_j R_j$ will be the reduced $\overline{\lambda\text{-}n}$-subsemigroups of S ([3], Theorem 2), where the union is disjoint; we also have that $R = T \cup E$, where $T = \{x \in S \mid x^{n+1} \neq x,\ x^{n+1} = e_j$ for some $j \in J$, $x = z^{n+1}$ for no $z \in S\}$.

Lemma. Let $x_n \in S(e_j)$ and at least one of $x_0, x_1, \ldots, x_n$ does not belong to R. Then:

(1) $[x_0 x_1 \cdots x_n] = e_j$ if $x_n \notin A_j$ or $x_k \in B_i \cup \{e_i\}$ for some $k = 0, 1, \ldots, n-1$;

(2) $[x_0 x_1 \cdots x_n] = e_j$ or x_n^{n+1} if $x_n \in A_j$ and, if $x_k \in S(e_i)$ then $x_k \in Q_i = A_i \cup (R_i \setminus \{e_i\})$ for every $k = 0, 1, \ldots, n-1$.

Proof. The assertion (2) follows from the fact that the product $[x_0 x_1 \cdots x_n]$ could take one of the values e_j, x_n^{n+1} and x_n and, because of $x_n \in A_j$, the last one is not to be taken into account ([3], Lema 7). Let $p = [x_0 x_1 \cdots x_n]$. Then $p \neq x_n$ since not all x_j belong to R ([3], Lemma 7). If $x_n \notin A_j$, then $x_n^{n+1} = e_j$, so, by Lemma 5 of [3], $p = e_j$. If $x_k = e_i$, $k < n$, then $p = [x_0 \cdots x_{k-1} e_i \cdots e_i q]$ where $q = [e_i \cdots e_i x_{k+1} \cdots \cdots x_n]$. If $q = e_j$, then by (1) of [3] we have that $p = e_j$, and if $q = x_n$, then $x_n = e_j$ ([3], Lemma 6) and previous applies. Let $q = x_n^{n+1}$; then $p = [[x_0 \cdots x_{k-1} e_i \cdots e_i x_n] x_n \cdots x_n]$. As above, the producr $[x_0 \cdots x_{k-1} e_i \cdots e_j x_n]$ can take one of the values e_j and x_n^{n+1}, and in both cases we have that $p = e_j$. Let us suppose that none x_k is an idempotent and that $x_k \in B_i$ for some $k = 0, 1, \ldots, n-1$. Then $x_k = y_i^{n+1}$, $y_i \in S(e_i)$; here y_i does not belong to R since, on the contrary, $x_k = e_i$ will be an idempotent, and this is a contradiction. Now, $p = [x_0 \cdots x_{k-1} y_i \cdots y_i r]$ where $r = [y_i \cdots y_i x_{k+1} \cdots x_n]$. Here $r \neq x_n$, since $y_i \notin R$. If $r = e_j$, then $p = e_j$, and if $r = x_n^{n+1}$, then $p = [[x_0 \cdots x_{k-1} y_i \cdots y_i x_n] x_n \cdots x_n]$ and for every of the two possible values e_j and x_n^{n+1} for $[x_0 \cdots x_{k-1} y_i \cdots y_i x_n]$ we have again that $p = e_j$. ∎

Before giving a description of the structure of an arbitrary $\overline{\lambda}$-n-semigroup, we shall construct a $\overline{\lambda}$-n-semigroup which we shall denote by (R, A, B, F, G). Let $\{(A_j, B_j, R_j, f_j, g_j) \mid j \in J\}$ be a family of pairwise disjoint unipotent $\overline{\lambda}$-n-semigroups and let us put $A = \bigcup_j A_j$, $B = \bigcup_j B_j$ and $R = \bigcup_j R_j$. Let R be a reduced $\overline{\lambda}$-n-semigroup and let $[\cdots]$ denotes the operation in R. Further, let $w \in J^n$ and for each pair (w, j) let f_{wj} be a mapping from $Q_w \times A_j$ into $\{0, 1\}$, where $Q_w = Q_{i_1} \times Q_{i_2} \times \cdots \times Q_{i_n}$ if $w = (i_1, i_2, \ldots i_n)$; if $w = (i, i, \ldots, i)$, then we write Q_i instead of Q_w. If the mappings f_{wj} have the properties $f_{jj}(x, x, \ldots, x) = 1$ for every $x \in A$ and $f_{wj}(x_0, x_1, \ldots, x_n) = 0$ if for some x_i, $i < n$, there exist $y_k \in R$, $k = 1, 2, \ldots, n$ such that $x_i = [y_1 \cdots y_n x_i]$, and if $g_j \colon A_j \to B_j$ are surjections for every $j \in J$, then in a similar way as in the case of unipotent $\overline{\lambda}$-n-semigroups, it can be shown that $S = A \cup B \cup R$ becomes a $\overline{\lambda}$-n-semigroup with respect to the operation $(\cdots)$ defined as follows:

$$(x_0 x_1 \cdots x_n) = \begin{cases} g_j(x_n) \text{ if } f_{wj}(x_0, x_1, \ldots, x_n) = 1 \\ [x_0 x_1 \cdots x_n] \text{ if all } x_k \in R \\ e_j \text{ in all other cases,} \end{cases}$$

where: $x_n \in S(e_j)$; $x_{r-1} \in Q_{i_r}$, $Q_{i_r} = A_{i_r} \cup (R_{i_r} \setminus \{e_{i_r}\})$, $r = 1, 2, \ldots, n$ and $w = (i_1, i_2, \ldots, i_n)$.

Conversely, if S is a $\overline{\lambda}$-n-semigroup, then by help of the lemma it can be shown that S can be obtained in the way described above. So, in summary, we can obtain the following description of the structure of a $\overline{\lambda}$-n-semigroup.

Theorem 4. *A* semigroup S is a $\overline{\lambda}$-n-semigroup if and only if $S = (R, A, B, F, G)$. ■

Note. The notion of λ-n-semigroup can be dualised as in [2] and all properties of [3] and this article about λ-n-semigroups dualy restated.

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n-ПОЛУГРУПИ СО ИДЕМПОТЕНТИ ВО КОИ СИТЕ n-ПОТПОЛУГРУПИ СЕ ЛЕВИ ИДЕАЛИ

Б. Трпеновски

(р е з и м е)

Во трудов се дава структурен опис за n-полугрупите што содржат идемпотенти и во кои сите n-потполугрупи се леви идеали. Најнапред се опишува структурата на унипотентните n-полугрупи со изнесената особина (Теорема 1), а потоа се опишуваат хомоморфизмите (Теорема 2) и изоморфизмите (Теорема 3) меѓу унипотентните n-полугрупи со особината од насловот. На крајот се дава опис на структурата и на произволните n-полугрупи со изнесената особина (Теорема 4).