1) By (1.10), if $x * y = z$, then $|z| > |x|$, $|z| > |y|$, and this implies that **R** satisfies (i).

2) If $x * x = y * y$, then (according to (1.7)) $x^2 = y^2$, and so $x = y$; thus (ii) holds.

Assume that $x * y = u * v$ and $\min\{[x], [y]\} = p \leq q = \min\{[u], [v]\}$. Then, by (0.5) and (1.5):

$$x^{(-p)} y^{(-p)} = \left(u^{(-q)} v^{(-q)}\right)^{(q-p)}. \tag{2.1}$$

If $p = q$, then $x^{(-p)} y^{(-p)} = u^{(-p)} v^{(-p)}$ which implies $x = u$, $y = v$.

If $p < q$, then by (2.1):

$$x^{(-p)} y^{(-p)} = \left(\left(u^{(-q)} v^{(-q)}\right)^{(q-p-1)}\right)^2,$$

which implies $x^{(-p)} = y^{(-p)}$, i.e. $x = y$.

Thus we have:

3) $x * y = u * v, \quad x \neq y, \quad u \neq v \Rightarrow x = u, \quad y = v$, i.e. (iii) is satisfied.

Finally, assume:

4) $x * x = y * z$, $y \neq z$.

Then, if $q = \min\{[y], [z]\}$, by (0.5) and (1.7), we have $x^2 = \left(y^{(-q)} z^{(-q)}\right)^{(q)}$, i.e.

$$x = \left(y^{(-q)} z^{(-q)}\right)^{(q-1)} = y^{(-1)} * z^{(-1)}.$$

Thus: $x = u * v$, $y = u^2$, $z = v^2$, where $u = y^{(-1)}, v = z^{(-1)}$. □

Now we will show the following

Lemma 2.2. Let $\mathbf{G} = (G, \cdot)$ be a groupoid which satisfies the condition (i) of *Th.* 2 and the following one:

(v) The set div(a) of divisors of an arbitrary element $a \in G$ is finite.

Then, for arbitrary $a \in G$, the set of lengths of divisor chains with the first member a is bounded.

(We denote by $L(a)$ the largest member of this set, and we say that $L(a)$ is the **length** of a.)

Proof. Consider the oriented graph of which the nodes are the elements of G, and for $a, b \in G$ there exists an edge with the initial node a and the terminal