(iv) $x^2 = yz,\ y \neq z \Rightarrow (\exists u,\ v)\ (x = uv,\ y = u^2,\ z = v^2)$.

Then the set P of primes in $\mathbf{H}$ is nonempty and the unique basis of $\mathbf{H}$.

Theorem 3. If $\mathbf{H}$ is a $\mathcal{V}$–free groupoid, then there exist subgroupoids $\mathbf{G}$, $\mathbf{Q}$ of $\mathbf{H}$ such that $\mathbf{G}$ is not $\mathcal{V}$–free, and $\mathbf{Q}$ is $\mathcal{V}$–free with an infinite rank.

The next results concern a sequence of functors in the variety $\mathcal{V}$. Namely, if $\mathbf{G}$ is a groupoid and k is a nonnegative integer, then we define the groupoid $\mathbf{G}^{(k)} = (G,\ (k))$ as follows:

$$x\ (k)\ y = (xy)^{(k)}. \tag{0.6}$$

(Note that the same symbol (k) is used in (0.6) with two different meanings: as an operation of G on the left, and as a transformation on the right side.)

Theorem 4. If $\mathbf{H}$ is a $\mathcal{V}$–free groupoid and $k \geq 1$, then:

1) $\mathbf{H}^{(k)} \in \mathcal{V}$

2) $\mathbf{H}^{(k)}$ is not $\mathcal{V}$–free, and

3) The subgroupoid $\mathbf{Q}$ of $\mathbf{H}^{(k)}$ generated by the basis B of $\mathbf{H}$ is a $\mathcal{V}$–free groupoid with the basis B.

In §i, $1 \leq i \leq 4$, we prove *Th. i*, and in §5 we consider the law $(xy)^n = x^ny^n$, where $n \geq 3$.

1. *A canonical description of $\mathcal{V}$–free groupoids*

First we will introduce a norm of the elements of F and state some lemmas in order to prove *Th.* 1. (As was mentioned in Introduction, we denote by B the basis of a given free groupoid $\mathbf{F} = (F,\ \cdot)$.)

The **norm** in F is defined as the homomorphism $u \mapsto |u|$ from $\mathbf{F}$ into the additive groupoid of positive integers, which is an extension of the mapping $B \to \{1\}$. Thus:

$$|vw| = |v| + |w|, \qquad |b| = 1, \tag{1.1}$$

for all $v,\ w \in F,\ b \in B$.

In the proof of *Th.* 1 we will use some of the following relations, where $u, v \in F$ and k, m are integers.

$$u^{(k)} \in F \Leftrightarrow k + [u] \geq 0 \tag{1.2}$$

$$k + [u] \geq 0 \Rightarrow \left|u^{(k)}\right| = 2^k|u| \tag{1.3}$$