Math. Maced.
Vol. 7 (2009)
29–37

COMPLETELY GENERALIZED SEMI-PRE CONTINUOUS MAPPINGS IN INTUTITIONISTIC FUZZY TOPOLOGICAL SPACES

D. JAYANTHI

Abstract. In this paper we introduce intuitionistic fuzzy completely generalized semi-pre continuous mappings. We investigate some of its properties. Also we provide some characterization of intuitionistic fuzzy completely generalized semi-pre continuous mappings.

1. Introduction

After the introduction of fuzzy sets by Zadeh [13], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. Intuitionistic fuzzy semi-pre continuous mappings in intuitionistic fuzzy topological spaces are introduced by Young Bae Jun and Seok- Zun Song [12]. R. Santhi and D. Jayanthi [8] introduced intuitionistic fuzzy generalized semi-pre continuous mappings. In this paper we introduce the notion of intuitionistic fuzzy completely generalized semi-pre continuous mappings and studied some of its properties. We provide some characterizations of intuitionistic fuzzy completely generalized semi-pre continuous mappings.

2. Preliminaries

Definition 2.1. [1] *An intuitionistic fuzzy set (IFS in short) A in X is an object having the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / xX\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2. [1] *Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then*
(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

Key words and phrases. Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi-pre $T_1/2$ space, intuitionistic fuzzy completely generalized semi-pre continuous mappings.

(b) $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$
(c) $A^c = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle\ / x \in X\}$.
The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3. [11] *The IFS* $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta}\rangle$ *where* $\alpha \in (0, 1], \beta \in [0, 1)$ *and* $\alpha + \beta \leq 1$ *is called an intuitionistic fuzzy point (IFP for short) in* X*.*

Definition 2.4. [11] *Two IFSs are said to be q-coincident (A_qB in short) if and only if there exists an element* $x \in X$ *such that* $\mu_A(x) > \nu_B(x)$ *or* $\nu_A(x) < \mu_B(x)$*.*

Definition 2.5. [3] *An intuitionistic fuzzy topology (IFT for short) on* X *is a family* τ *of IFSs in* X *satisfying the following axioms.*

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$
(iii) $\bigcup G_i \in \tau$ *for any family* $\{G_i / i \in J\} \subseteq \tau$*.*

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.6. [4] *Let* (X, τ) *be an IFTS and* $A = \langle x, \mu_A, \nu_A\rangle$ *be an IFS in* X*. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by*
$int(A) = \bigcup\{G / G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$
$cl(A) = \bigcap\{K / K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$

Note that for any IFS A in (X, τ), we have $\mathrm{cl}(A^c) = [int(A)]^c$ and $\mathrm{int}(A^c) = [cl(A)]^c$[11].

Definition 2.7. [6] *An IFS* $A = \langle x, \mu_A, \nu_A\rangle$ *in an IFTS* (X, τ) *is said to be an*

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$
(ii) intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$
(iii) intuitionistic fuzzy α *closed set (IFαCS in short) if* $cl(int(cl(A)) \subseteq A$*.*

The respective complements of the above IFCSs are called their respective IFOSs.

Definition 2.8. [12] *An IFS* $A = \langle x, \mu_A, \nu_A\rangle$ *in an IFTS* (X, τ) *is said to be an*

(i) intuitionistic fuzzy semi-pre closed set (IFSPCS for short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$*.*
(ii) intuitionistic fuzzy semi-pre open set (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short) B such that $B \subseteq A \subseteq cl(B)$*.*

Note that an IFS A is an IFSPCS if and only if $int(cl(int(A))) \subseteq A$ and an IFSPOS if and only if $A \subseteq cl(int(cl(A)))$ [7].

Definition 2.9. [7] *Let A be an IFS in an IFTS* (X, τ). *Then*
$spint(A) = \bigcup G/G$ *is an IFSPOS in* X *and* $G \subseteq A$.
$spcl(A) = \bigcap K/K$ *is an IFSPCS in* X *and* $A \subseteq K$.

Note that for any IFS A in (X, τ), we have $spcl(A^c) = [spint(A)]^c$ and $spint(Ac) = [spcl(A)]c$ [7].

Definition 2.10. [11] *An IFS Ais an*

(i) intuitionistic fuzzy regular closed set (IFRCS for short) if $A = cl(int(A))$.
(ii) intuitionistic fuzzy generalized closed set (IFGCS for short) if $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is an IFOS.*

Definition 2.11. [7] *An IFS A in an IFTS* (X, τ) *is said to be an intuitionistic fuzzy generalized semi-pre closed set (IFGSPCS for short) if* $spcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is an IFOS in* (X, τ).

Every IFCS, IFGCS, IFSCS, IFPCS, IFRCS, IFαCS and IFSPCS is an IFGSPCS but the separate converses may not be true in general [7].

Definition 2.12. [7] *The complement* A^c *of an IFGSPCS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy generalized semi-pre open set (IFGSPOS for short) in* X.

Every IFOS, IFGOS, IFSOS, IFPOS, IFROS, IFαOS and IFSPOS is an IFGSPOS but the separate converses may not be true in general [7].

Definition 2.13. [6] *Let* f *be a mapping from an IFTS* (X, α) *into an IFTS* (Y, σ). *Then* f *is said to be an intuitionistic fuzzy continuous (IF continuous for short) mapping if* $f^{-1}(B) \in IFO(X)$ *for every* $B \in \sigma$

Definition 2.14. [6] *Let* f *be a mapping from an IFTS* (X, τ) *into an IFTS* (Y, σ). *Then* f *is said to be an*

(i) intuitionistic fuzzy semi continuous (IFS continuous for short) mapping if $f^{-1}(B) \in IFSO(X)$ *for every* $B \in \sigma$
(ii) intuitionistic fuzzy α *- continuous (IF*α *- continuous for short) mapping if* $f^{-1}(B) \in IF\alpha O(X)$ *for every* $B \in \sigma$
(iii) intuitionistic fuzzy pre continuous (IFP continuous) mapping if $f^{-1}(B) \in IFPO(X)$ *for every* $B \in \sigma$

Definition 2.15. [10] *Let* $c(\alpha, \beta)$ *be an IFP of an IFTS* (X, τ). *An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of* $c(\alpha, \beta)$ *if there exists an IFOS B in X such that* $c(\alpha, \beta) \in B \subseteq A$.

Definition 2.16. [11] *Let f be a mapping from an IFTS* (X, τ) *into an IFTS* (Y, σ). *Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous for short) mapping if* $f^{-1}(B) \in IFGC(X)$ *for every* $IFCSB$ *in* Y.

Every IF continuous mapping is an IFG continuous mapping but the converse may not be true in general [11].

Definition 2.17. [12] *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy semi-pre continuous (IFSP continuous for short) mapping if $f^{-1}(B) \in IFSPO(X)$ for every $B \in \sigma$*

Every IFS continuous mapping and IFP continuous mappings are IFSP continuous mapping but the converses may not be true in general[12].

Definition 2.18. [8] *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy semi-pre irresolute (IFSP irresolute for short) mapping if $f^{-1}(B)$ is an IFSPCS of X for every IFSPCS B in Y.*

Definition 2.19. [8] *If every IFGSPCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an intuitionistic fuzzy semi- pre $T_{1/2}$ space (IFSP$T_{1/2}$ space for short).*

Definition 2.20. [8] *A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy generalized semi-pre continuous (IFGSP continuous for short) mapping if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFCS V of (Y, σ).*

Definition 2.21. [9] *An IFS A is said to be an intuitionistic fuzzy dense (IFD for short) in another IFS B in an IFTS (X, τ), if cl(A) = B.*

Definition 2.22. [5] *Let X and Y be two IFTSs. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle / y \in Y\}$ be IFSs of X and Y respectively. Then $A \times B$ is an IFS of $X \times Y$ defined by*
$(A \times B)(x, y) = \langle (x, y), \min(\mu_A(x), \mu_B(y)), \max(\gamma_A(x), \gamma_B(y))\rangle$.

Definition 2.23. [5] *Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for every $(x_1, x_2) \in X_1 \times X_2$.*

3. Intuitionistic fuzzy completely generalized semi-pre continuous mappings

In this section we introduce intuitionistic fuzzy completely generalized semi-pre continuous mappings and study some of their properties.

Definition 3.1. *A mapping $f : X \to Y$ is said to be an intuitionistic fuzzy completely generalized semi-pre continuous mapping (IF completely GSP continuous mapping for short) if $f^{-1}(V)$ is an IFRCS in X for every IFGSPCS V in Y.*

Theorem 1. *Every IF completely GSP continuous mapping is an IFGSP continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely GSP continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFGSPCS, V is an IFGSPCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping. □

For the sake of simplicity, we shall use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of $A = x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)$ in the following examples.

Similarly we shall use the notation $B = \langle y, (\mu_u, \mu_v), (\nu_u, \nu_v)\rangle$ instead of $B = \langle y, (u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following examples.

Example 3.1. *Let* $X = \{a, b\}$, $Y = \{u, v\}$ *and* $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)$ *and* $G_4 = \langle y, (0.5_u, 0.5_v), (0.5_u, 0.4_v)\rangle$. *Then* $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ *and* $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ *are IFTs on* X *and* Y *respectively. Define a mapping* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = u$ *and* $f(b) = v$. *Then* f *is an IFGSP continuous mapping but not an IF completely GSP continuous mapping, since* G_3^c *is an IFGSPCS in* Y *but* $f^{-1}(G_3^c) = \langle x, (0.8_a, 0.7_b), (0.2_a, 0.3_b)\rangle$ *is not an IFRCS in* X. *Since* $cl(int(f^{-1}(G_3^c))) = G_1^c \neq f^{-1}(G_3^c)$.

Theorem 2. *Every IF completely GSP continuous mapping is an IFaGSP continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely GSP continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFGSPCS, V is an IFGSPCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFGSPCS, $f^{-1}(V)$ is an IFGSPCS in X. Hence f is an IFaGSP continuous mapping. □

Example 3.2. *In example 3.1, the mapping is an IFaGSP continuous mapping but not an IF completely GSP continuous mapping.*

Theorem 3. *Every IF completely GSP continuous mapping is an IF continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely GSP continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFGSPCS, V is an IFGSPCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFCS, $f^{-1}(V)$ is an IFCS in X. Hence f is an IF continuous mapping. □

Example 3.3. *Let* $X = \{a, b\}, Y = \{u, v\}$ *and* $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$, *Then* $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ *and* $\sigma = \{0_\sim, G_3, 1_\sim\}$ *are IFTs on* X *and* Y *respectively. Define a mapping* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = u$ *and* $f(b) = v$. *Then* f *is an IF continuous mapping but not an IF completely GSP continuous mapping, since* G_3^c *is an IFGSPCS in* Y *but* $f^{-1}(G_3^c) = \langle x, (0.8_a, 0.7_b), (0.2_a, 0.3_b)\rangle$ *is not an IFRCS in* X, *since* $cl(int(f^{-1}(G_3^c))) = G_1^c \neq f^{-1}(G_3^c)$.

Theorem 4. *Every IF completely GSP continuous mapping is an IFS continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely GSP continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFGSPCS, V is an IFGSPCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFSCS, $f^{-1}(V)$ is an IFSCS in X. Hence f is an IFS continuous mapping. □

Example 3.4. *In Example 3.3,* f *is an IFS continuous mapping but not an IF completely GSP continuous mapping.*

Theorem 5. *Every IF completely GSP continuous mapping is an IFG continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely GSP continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFGSPCS, V is an IFGSPCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFGCS, $f^{-1}(V)$ is an IFGCS in X. Hence f is an IFG continuous mapping. □

Example 3.5. *In Example 3.1, the mapping is an IFG continuous mapping but not an IF completely GSP continuous mapping.*

Theorem 6. *Every IF completely GSP continuous mapping is an IFP continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely GSP continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFGSPCS, V is an IFGSPCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFPCS, $f^{-1}(V)$ is an IFPCS in X. Hence f is an IFP continuous mapping. □

Example 3.6. *In Example 3.3, the mapping is an IFP continuous mapping but not an IF completely GSP continuous mapping.*

Theorem 7. *Every IF completely GSP continuous mapping is an IFSP continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely GSP continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFGSPCS, V is an IFGSPCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFSPCS, $f^{-1}(V)$ is an IFSPCS in X. Hence f is an IFSP continuous mapping. □

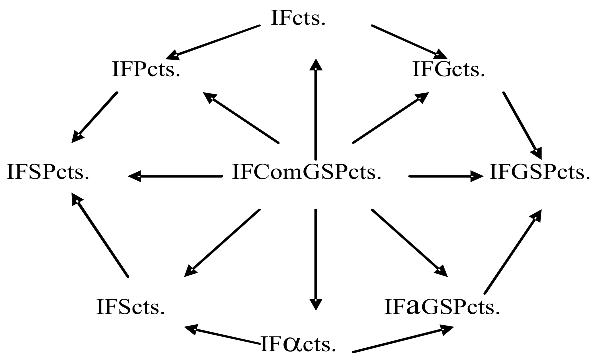
Example 3.7. *In Example 3.3, the mapping is an IFSP continuous mapping but not an IF completely GSP continuous mapping.*

Theorem 8. *Every IF completely GSP continuous mapping is an IFα continuous mapping but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF completely GSP continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFGSPCS, V is an IFGSPCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFαCS, $f^{-1}(V)$ is an IFαCS in X. Hence f is an IFα continuous mapping. □

Example 3.8. *In Example 3.3, the mapping is an IFα continuous mapping but not an IF completely GSP continuous mapping.*

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram cts. means continuous and IFcomGSPcts. means IF completely GSP continuous.



The reverse implications are not true in general in the above diagram.

Theorem 9. *Let $c(\alpha,\beta)$ be an IFP in an IFTS (X,τ). A mapping $f : X \to Y$ is an IF completely GSP continuous mapping if for every IFGSPOS A in Y with $f(c(\alpha,\beta)) \in A$, there exists an IFROS B in X with $c(\alpha,\beta) \in B$ such that $f^{-1}(A)$ is IFD in B.*

Proof. Let A be an IFGSPOS in Y and let $f(c(\alpha,\beta)) \in A$. Then there exists an IFROS B in X with $c(\alpha,\beta) \in B$ such that $cl(f^{-1}(A)) = B$. Since B is an IFROS, $cl(f^{-1}A))$ is also an IFROS in X. Therefore $int(cl(cl(f^{-1}(A)))) = cl(f^{-1}A))$. That is $int(cl(f^{-1}(A))) = cl(f^{-1}A))$. This implies $f^{-1}(A)$ is also an IFROS in X. □

Theorem 10. *If $f : X \to Y$ is an IF completely GSP continuous mapping where X is an IFSP$T_{1/2}$ space, then $spcl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every IFSPOS $A \subseteq Y$.*

Proof. Let A be an IFSPOS in Y. Then $cl(A)$ is an IFRCS in Y. Hence $cl(A)$ is an IFGSPCS in Y. By hypothesis, $f^{-1}(cl(A))$ IFRCS in X and thus an IFSPCS in X. Therefore $spcl(f^{-1}(A)) \subseteq spcl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. □

Corollary 10.1. *If $f : X \to Y$ is an IF completely GSP continuous mapping where X is an IFSP$T_{1/2}$ space, then $spcl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every IFSOS $A \subseteq Y$.*

Proof. Since every IFSOS is an IFSPOS, the proof directly follows from the Theorem. □

Theorem 11. *A mapping $f : X \to Y$ is an IF completely GSP continuous mapping if and only if $f^{-1}(V)$ is an IFROS in X for every IFGSPOS V in Y.*

Proof. Straightforward. □

Theorem 12. *Let $f : X \to Y$ be a mapping. Then the following are equivalent.*

(i) f is an IF completely GSP continuous mapping.

(ii) $f^{-1}(V)$ *is an IFROS in* X *for every IFGSPOS* V *in* Y.
(iii) *for every IFP* $c(\alpha,\beta)\in X$ *and for every IFGSPOS* B *in* Y *such that* $f(c(\alpha,\beta))\in B$ *there exists an IFROS in* X *such that* $c(\alpha,\beta)\in A$ *and* $f(A)\subseteq B$.

Proof. $(i)\Rightarrow(ii)$ is obvious by Theorem
$(ii)\Rightarrow(iii)$ Let $c(\alpha,\beta)\in X$ and $B\subseteq Y$ such that $f(c(\alpha,\beta))\in B$. This implies $c(\alpha,\beta)\in f^{-1}(B)$. Since B is an IFGSPOS in Y, by hypothesis $f^{-1}(B)$ is an IFROS in X. Let $A=f^{-1}(B)$. Then $c(\alpha,\beta)\subseteq f^{-1}(f(c(\alpha,\beta)))\in f^{-1}(B)=A$. Therefore $c(\alpha,\beta)\in A$ and $f(A)=f(f^{-1}(B))\subseteq B$. This implies $f(A)\subseteq B$.
$(iii)\Rightarrow(i)$ Let $B\subseteq Y$ be an IFGSPOS. Let $c(\alpha,\beta)\in X$ and $f(c(\alpha,\beta))\in B$. By hypothesis, there exists an IFROS C in X such that $c(\alpha,\beta)\in C$ and $f(C)\subseteq B$. This implies $C\subseteq f^{-1}(f(C))\subseteq f^{-1}(B)$. Therefore $c(\alpha,\beta)\in C\subseteq f^{-1}(B)$. That is $f^{-1}(B)=\bigcup_{c(\alpha,\beta)\in f^{-1}(B)}c(\alpha,\beta)\subseteq\bigcup_{c(\alpha,\beta)\in f^{-1}(B)}C\subseteq f^{-1}(B)$. This implies $f^{-1}(B)=\bigcup_{c(\alpha,\beta)\in f^{-1}(B)}C$. Since union of IFROSs is IFRO, $f^{-1}(B)$ is an IFROS in X. Hence f is an IF completely GSP continuous mapping. □

Theorem 13. *A mapping* $f:X\to Y$ *is an IF completely GSP continuous mapping if for every IFP* $c(\alpha,\beta)\in X$ *and for every IFN* A *of* $f(c(\alpha,\beta))$, *there exists an IFROS* $B\subseteq X$ *such that* $c(\alpha,\beta)\in B\subseteq f^{-1}(A)$.

Proof. Let $c(\alpha,\beta)\in X$ and let A be an IFN of $f(c(\alpha,\beta))$. Then there exists an IFOS C in Y such that $f(c(\alpha,\beta))\in C\subseteq A$. Since every IFOS is an IFGSPOS, C is an IFGSPOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $c(\alpha,\beta)\in f^{-1}(C)$. Now let $f^{-1}(C)=B$. Therefore $c(\alpha,\beta)\in B\subseteq f^{-1}(A)$. □

Theorem 14. *A mapping* $f:X\to Y$ *is an IF completely GSP continuous mapping if for every IFP* $c(\alpha,\beta)\in X$ *and for every IFN* A *of* $f(c(\alpha,\beta))$, *there exists an IFROS* $B\subseteq X$ *such that* $c(\alpha,\beta)\in B$ *and* $f(B)\subseteq A$.

Proof. Let $c(\alpha,\beta)\in X$ and let A be an IFN of $f(c(\alpha,\beta))$. Then there exists an IFOS C in Y such that $f(c(\alpha,\beta))\in C\subseteq A$. Since every IFOS is an IFGSPOS, C is an IFGSPOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $c(\alpha,\beta)\in f^{-1}(C)$. Now let $f^{-1}(C)=B$. Therefore $c(\alpha,\beta)\in B\subseteq f^{-1}(A)$. Thus $f(B)\subseteq f(f^{-1}(A))\subseteq A$. That is $f(B)\subseteq A$. □

Theorem 15. *A mapping* $f:X\to Y$ *is an IF completely GSP continuous mapping then* $int(cl(f^{-1}(int(B))))\subseteq f^{-1}(B)$ *for every IFS* B *in* Y.

Proof. Let $B\subseteq Y$ be an IFS. Then $int(B)$ is an IFOS in Y and hence an IFGSPOS in Y. By hypothesis, $f^{-1}(int(B))$ is an IFROS in X. Hence $int(cl(f^{-1}(int(B))))=f^{-1}(int(B))\subseteq f^{-1}(B)$. □

Theorem 16. *For any two IF completely GSP continuous functions* $f_1,f_2:(X,\tau)\to(Y,\sigma)$, *the function* $(f_1,f_2):(X,\tau)\to(Y\times Y,\sigma\times\sigma)$ *is also an IF completely GSP continuous function where* $(f_1,f_2)(x)=(f_1(x),f_2(x))$ *for every* $x\in X$.

Proof. Let $A\times B$ be an IFGSPOS in $Y\times Y$. Then $(f_1,f_2)^{-1}(A\times B)(x)=(A\times B)(f_1(x),f_2(x))=\langle x,\min(\mu_A(f_1(x)),\mu_B(f_2(x))),\max(\gamma_A(f_1(x)),\gamma_B(f_2(x)))\rangle=$

$\langle x, \min(f_1^{-1}(\mu_A(x)), f_2^{-1}(\mu_B(x))), \max(f^1-1(\gamma_A(x)), f_2^{-1}(\gamma_B(x)))\rangle = f_1^{-1}(A) \cap f_2^{-1}(B)(x)$. Since f_1 and f_2 are IF completely GSP continuous functions, $f_1^{-1}(A)$ and $f_2^{-1}(B)$ are IFROSs in X. Since intersection of IFROSs is an IFROS, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X. Hence (f_1, f_2) is an IF completely GSP continuous mappings. □

Theorem 17. *A mapping $f : X \to Y$ is an IF completely GSP continuous mapping then the following are equivalent.*

(i) for any IFGSPOS A in Y and for any IFP $c(\alpha, \beta) \in X$, if $f(c(\alpha, \beta))_q A$ then $c(\alpha, \beta)_q int(f^{-1}(A))$.

(ii) For any IFGSPOS A in Y and for any $c(\alpha, \beta) \in X$, if $f(c(\alpha, \beta))_q A$ then there exists an IFOS B such that $c(\alpha, \beta)_q B$ and $f(B) \subseteq A$.

Proof. $(i) \Rightarrow (ii)$ Let $A \subseteq Y$ be an IFGSPOS and let $c(\alpha, \beta) \in X$. Let $f(c(\alpha, \beta))_q A$. Then $c(\alpha, \beta)_q f^{-1}(A)$. (i) implies that $c(\alpha, \beta)_q int(f^{-1}(A))$, where $int(f^{-1}(A))$ is an IFOS in X. Let $B = int(f^{-1}(A))$. Since $int(f^{-1}(A)) \subseteq f^{-1}(A)$, $B \subseteq f^{-1}(A)$. Then $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

$(ii) \Rightarrow (i)$ Let $A \subseteq Y$ be an IFGSPOS and let $c(\alpha, \beta) \in X$. Suppose $f(c(\alpha, \beta))_q A$, then by (ii) there exists an IFOS B in X such that $c(\alpha, \beta)_q B$ and $f(B) \subseteq A$. Now $B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$. That is $int(B) \subseteq int(f^{-1}(A))$. Therefore $c(\alpha, \beta)_q B$ implies $c(\alpha, \beta)_q int(f^{-1}(A))$. □

References

[1] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, (1986), 87–96.

[2] C. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., (1968), 182- 190.

[3] D. Coker, *An introduction to intuitionistic fuzzy topological space*, Fuzzy sets and systems, (1997), 81-89.

[4] H. Gurcay, D. Coker and Es. A. Haydar, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, The J. fuzzy mathematics, (1997), 365-378.

[5] I. M. Hanafy and El-Arish, *Completely continuous functions in intuitionistic fuzzy topological spaces*, Czechoslovak Mathematical journal, (2003), 793-803.

[6] Joung Kon Jeon, Young Bae Jun and Jin Han Park, *Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy pre continuity*, Int. J. Math. and Math. Sci, (2005), 3091-3101.

[7] R. Santhi and D. Jayanthi, *Intuitionistic fuzzy generalized semi-pre closed sets*, Tripura Math. Soci., (2009), 61- 72.

[8] R. Santhi and D. Jayanthi, I*ntuitionistic fuzzy generalized semi-pre continuous mappings*, Int. J. Contemp.Math. Sciences, (2010), 1455-1469.

[9] R. Santhi and D. Jayanthi, *Intuitionistic fuzzy almost generalized semi-pre continuous mappings* (accepted by Tamkang Journal of Mathematics, Taiwan)

[10] Seok Jong Lee and Eun Pyo Lee, *The category of intuitionistic fuzzy topological spaces*, Bull. Korean Math. Soc. (2000), 63-76.

[11] S. S. Thakur and Rekha Chaturvedi, *Regular generalized closed sets in intuitionistic fuzzy topological spaces*, Universitatea Din Bacau,Studii Si Cercetari Stiintifice, Seria: Matematica, (2006), 257-272.

[12] Young Bae Jun and Seok- Zun Song, *Intuitionistic fuzzy semi-pre open sets and Intuitionistic semi-pre continuous mappings*, Jour. of Appl. Math & computing, (2005), 467-474.

[13] L. A. Zadeh, *Fuzzy sets*, Information and control, (1965), 338-353.

Department of Mathematics, NGM College, Pollachi, Tamil Nadu.
E-mail address: jayanthimaths@rediffmail.com