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ON A CONJECTURE OF P. NYLEN AND L. RODMAN

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Abstract. In this note we prove that conjecture of P. Nylen and L. Rodman [2, Integral Equations and Operator Theory, Vol. 13(1990), 728-749] is true.

We use the same notations as in [2], where there is the following

Conjecture [2, Conjecture 5.11]. Every Banach algebra A (with unit) has the spectral radius property, i.e. for every $a \in A$ with the spectral point sequence $\{\mu_j(a)\}_{j=1}^{\infty}$ of infinite length satisfying

$$\mu_m(a) = \lim_{n\to\infty} \mu_n(a)$$

for some integer m, the limit $\lim_{n\to\infty} \mu_n(a)$ is actually equal to the spectral radius of a in the factor algebra A/K, where K is the norm closure of the ideal of finite rank elements.

To verify [2, Conjecture 5.11] it is enough to prove

Theorem. Let A be a complex Banach algebra with unit 1 and K be the closure of the ideal F of finite rank elements of A. If $a \in A$, $\lambda \in \sigma(a)$ and $|\lambda| > r_k(a)$, then λ is a f.m. spectral point of a.

Proof. Set $A' = A/\mathrm{rad}(A)$, where rad(A) is the Jacobson radical of A. The algebra A' is semisimple and so the socle of A', soc(A'), exist. We write x' for the coset x+rad(A) and if $S \subset A$ write $S' = \{x' : x \in S\}$.

Suppose that $a \in A$, $\lambda \in \sigma(a)$ and $|\lambda| > r_k(a)$. F' is a two-sided ideal of A', and from [2, Corollary 2.3] and [3, Theorem 3.2], it

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follows that $F' \subset soc(A')$. It is easy to see that $r_k(a) \geq r_{cl(F')}(a'+cl(F')) \geq r_{cl(soc(A'))}(a'+cl(soc(A')))$, where cl(F')) and cl(soc(A')) denote, respectively, the closure of F' and soc(A'). Now, according to [1,F.3], it follows that $a-\lambda$ is a Fredholm element of A. From [1, Theorem F.3.7, F.3.8 and F.3.9], it follows that λ is an isolated point in $\sigma(a)$. Let $e_\lambda(a)$ be the Riesz idempotent associated with λ. From $|\lambda| > r_k(a)$, it follows that $e_\lambda(a)+K=K$, and by [3, Theorem 4.6] we conclude that $e_\lambda(a) \in F$. □

R E F E R E N C E S

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ЗА ЕДНО ТВРДЕЊЕ НА P. NYLEN И L. RODMAN

Владимир Ракочевиќ

Р е з и м е

Овде докажуваме дека хипотезата на P. Nilen и L. Rodman [2] е точна.

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