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ANALYSIS OF A PERTURBED HAMILTONIAN SYSTEM

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Abstract

In this paper we study the existence, number and distribution of limit cycles of the perturbed Hamiltonian system given by:

$$x' = 4y\left(abx^2 - by^2 + 1\right) + \varepsilon x(ux^n + vy^n - b\frac{\beta+1}{\mu+1}x^{\mu}y^{\beta} - ux^2 - \lambda)$$

$$y' = 4x\left(ax^2 - aby^2 - 1\right) + \varepsilon y(ux^n + vy^n + bx^{\mu}y^{\beta} - vy^2 - \lambda)$$

where $\mu + \beta = n, 0 < a < b < 1, 0 < \varepsilon \ll 1$, u, v, λ are the real parameters and $n = 2k$, k integer positive.

Abelian integral method is employed, [15]. For some particular parameters we draw the distribution limit cycle diagrams.

1. Introduction

The present work deals with the existence, number and distribution of limit cycles of a polynomial differential system of a given degree. This problem is still unsolved even for the quadratic polynomial differential systems. As long as we know, there are four methods which are used for studying the number of limit cycles bifurcating from a center. The first one is based on the Poincaré return map [4], the second on the Poincaré-Melnikov integral method [14], the third on the Abelian integral method [15] and the last is presented in [16]. In the plane, the first two are equivalent.

It is known that one way to produce limit cycles is by perturbing an Hamiltonian system which has one or more centers, in such a way that limit cycles bifurcate in the perturbed system from some of the periodic orbits in the original system.

The following result is registered in [2].

Theorem 1.1. *Consider the perturbed Hamiltonian system*

$$x' = -\frac{\partial H}{\partial y} + P(x,y,\alpha), \quad y' = \frac{\partial H}{\partial x} + Q(x,y,\alpha). \tag{1}$$

Assume that $P(x,y,0) = Q(x,y,0) = 0$, *the curve* Γ^h *defined by Hamiltonian* $H(x,\theta) = h$ *of system* (1) *is a periodic orbit that extends outside as* h *increases, and* $\Gamma^h(D)$ *is the area inside* Γ^h. *If there exists* h_0 *such that function*

$$A(h) = \iint\limits_{\Gamma^h(D)} [P''_{x\alpha}(x,y,0) + Q''_{y\alpha}(x,y,0)]dxdy \tag{2}$$

satisfies $A(h_0) = 0, A'(h_0) \neq 0, \alpha A'(h_0) < 0(> 0)$, *then system* (1) *has only one stable (unstable) limit cycle nearby* Γ^{h_0} *for* α *very small. If* Γ^h *constricts inside as* h *increases , the stability of the limit cycle is opposite with above. If* $A(h) \neq 0$, *then system* (1) *has no limit cycle.*

The integral $A(h)$ is called the *Abelian integral* and the problem is known as the weakened 16-th Hilbert problem.

If the form of the system (1) is:

$$\begin{cases} x'(t) = -\dfrac{\partial H}{\partial y} + \varepsilon x(p(x,y) - \lambda), \\ y'(t) = \dfrac{\partial H}{\partial x} + \varepsilon y(q(x,y) - \lambda)\,, \end{cases} \tag{3}$$

where $p(0,0) = q(0,0) = 0$, then, by using the above Theorem 1.1, from $A(h) = 0$, we get:

$$\lambda = \lambda(h) = \frac{\iint\limits_{\Gamma^h(D)} f(x,y)dxdy}{2 \iint\limits_{\Gamma^h(D)} dxdy} \tag{4}$$

where $f(x,y) = xp'_x + yp'_y + p + q$.

This function $\lambda(h)$ is called *the detection function* of system (3).

From the Theorem 1.1 and by using the detection function $\lambda(h)$ we get the following result :

Proposition 1.1. a) *If* $(h_0, \lambda(h_0))$ *is an intersecting point of line* $\lambda = \lambda_0$ *and the detection curve* $\lambda = \lambda(h)$, *and* $\lambda'(h_0) > 0(< 0)$, *then system* (3) *has only one stable (unstable) limit cycle nearby* Γ^{h_0} *when* $\lambda = \lambda_0$; b) *If line* $\lambda = \lambda_0$ *and the detection curve* $\lambda = \lambda(h)$ *have no intersecting point, then the system* (3) *has no any limit cycle when* $\lambda = \lambda_0$. *If the* Γ^h

constricts inside as h increasing , the stability of the limit cycle is opposite with above.

The perturbed Hamiltonian system (5) considered in this paper is the following:

$$\begin{cases} x' = 4y\left(-by^2 + abx^2 + 1\right) + \varepsilon x(p(x,y) - \lambda) \\ y' = 4x\left(ax^2 - aby^2 - 1\right) + \varepsilon y(q(x,y) - \lambda) \end{cases} \tag{5}$$

where
$p(x,y) = ux^n + vy^n - b\frac{\beta+1}{\mu+1}x^\mu y^\beta - ux^2$, $q(x,y) = ux^n + vy^n + bx^\mu y^\beta - vy^2$, $\beta + \mu = n$, $0 < a < b < 1, 0 < \varepsilon \ll 1$, u, v, λ are the real parameters and $n = 2k$, k integer positive. Some results on this system are reported in [12], [13] and on a related system in [11].

This paper is organized as follows. In Section 2, we study the global portrait of the unperturbed system. In Section 3, we gain both analytical forms and numerical results of the detection functions. Finally, we obtain the distribution of the limit cycles.

2. The behavior of the unperturbed system

The unperturbed system (6) corresponding to system (5) is

$$\begin{cases} x' = 4y\left(-by^2 + abx^2 + 1\right) \\ y' = 4x\left(ax^2 - aby^2 - 1\right) \end{cases} \tag{6}$$

that is, system (5) in the case $\varepsilon = 0$.

System (6) has nine finite singular points and they are:

$$O(0,0), A_1\left(\sqrt{\frac{1+a}{a(1-ba)}}, \frac{1}{b-b^2a}\sqrt{b(1-ba)(1+b)}\right),$$

$$A_2\left(\sqrt{\frac{1+a}{a(1-ba)}}, \frac{-1}{b-b^2a}\sqrt{b(1-ba)(1+b)}\right),$$

$$A_3\left(-\sqrt{\frac{1+a}{a(1-ba)}}, \frac{1}{b-b^2a}\sqrt{b(1-ba)(1+b)}\right),$$

$$A_4\left(-\sqrt{\frac{1+a}{a(1-ba)}}, \frac{-1}{b-b^2a}\sqrt{b(1-ba)(1+b)}\right),$$

$$B_1(0,\sqrt{\frac{1}{b}}), B_2(0,-\sqrt{\frac{1}{b}}), C_1(\sqrt{\frac{1}{a}},0) \text{ and } C_2(-\sqrt{\frac{1}{a}},0).$$

By computing the eigenvalues at each singular point we have that O, A_1, A_2, A_3, A_4 are centers while the other singular points B_1, B_2, C_1, C_2 are hyperbolic saddle points.

Further we have that the Hamiltonian of system (6) is

$$H(x,y) = -(ax^4 + by^4) + 2abx^2y^2 + 2\left(x^2 + y^2\right) = h \tag{7}$$

and

$$H(A_i) = \frac{2ba + b + a}{ba(1 - ba)},\ i = 1 - 4, \quad H(B_k) = \frac{1}{b}, H(C_k) = \frac{1}{a},\ k = 1, 2.$$

Because $0 < a < b < 1$ we get that: $H(O) < H(B_1) < H(C_1) < H(A_1)$

In polar coordinates, $x = r\cos\theta, y = r\sin\theta$, system (6) becomes:

$$r' = -r^3 p'(\theta), \quad \theta' = -1 + r^2 p(\theta) \tag{8}$$

and the Hamiltonian (7) reads

$$H(r,\theta) = -r^4 p(\theta) + 2r^2 = h, \tag{9}$$

where

$$p(\theta) = a\cos^4\theta + b\sin^4\theta - 2ab\cos^2\theta\sin^2\theta. \tag{10}$$

Remark 1. *The equilibrium points A_1, A_2, A_3, A_4 lie on the lines $d_\pm$:* $\theta = \pm\arccos\sqrt{\dfrac{b + ba}{a + b + 2ab}}$.

Theorem 2.1. *As h varies on the real line, the closed curves defined by Hamiltonian* (9) *can be divided as follows* [12]*:*

1. $\Gamma_1^h : -\infty < h < 0$, *this corresponds to an orbit that surrounds all critical points, fig.*1 *a).*

2. $\Gamma_2^h \cup \Gamma_1^h : 0 \leq h < \frac{1}{b}$, *this corresponds to an orbit* $\left(\Gamma_2^h\right)$ *that surrounds only the origin and a curve of type* $\left(\Gamma_1^h\right)$, *fig.*1 *b)-a).*

3. $\Gamma_3^h : \frac{1}{b} < h < \frac{1}{a}$, *this corresponds to two symmetric orbits that do not intersect the Oy axis but encircle the rest of critical points. If $h = \frac{1}{b}$ we get four heteroclinic orbits connecting the critical points B_1 and B_2, fig.*2 *b)-a).*

4. Γ_4^h : $\frac{1}{a} < h < \frac{2ba + b + a}{ba(1 - ba)}$, *this corresponds to four orbits that surround respectively the $A_i, i = 1 - 4$, equilibrium points. If $h = \frac{1}{a}$ we have four homoclinic orbits connecting the critical points C_1 and C_2, fig.*3

b)-a) . Note that as h increases, the curves Γ_1^h, Γ_3^h *and* Γ_4^h *shrink, while* Γ_2^h *extends.*

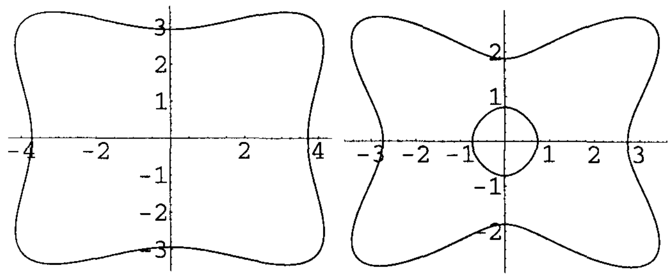

Fig. 1. Orbit of type a) L_1 (left) b) L_2 and L_1 (right)

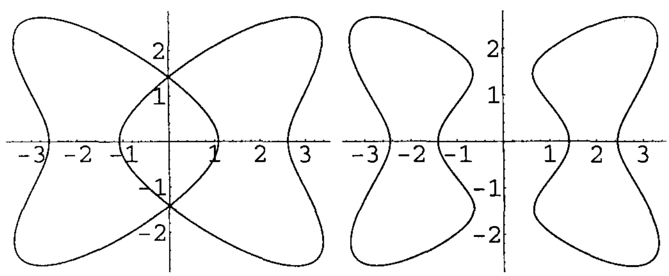

Fig. 2. a) Four heteroclinic orbits connecting two critical points B_1 and B_2 (left) b) Two orbits of type L_3 (right)

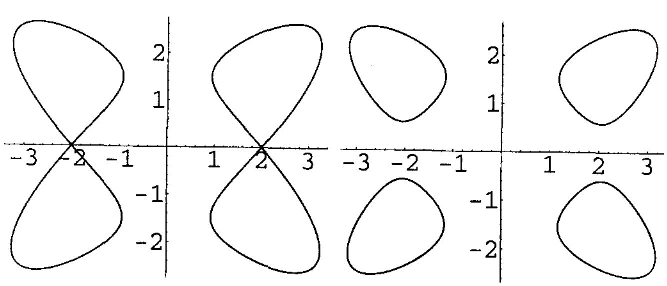

Fig. 3. a) Four homoclinic orbits connecting the critical points C_1 and C_2 (left) b) Four orbits of type L_4 (right)

3. Numerical explorations

In this section we numerically compute the detection curves and the distribution of limit cycles. The four detection functions can be computed numerically, and for a given h, they depend on u and v, (see tables 1-4). On the other hand, for two given values of u and v, the detection curves can be plotted on the (h, λ)-plane, as illustrated in fig.4. By the Proposition 1.1 and the detection function graphs, the existence, number and distribution of limit cycles can then be obtained. We consider here the case $n = 8$, that corresponds to perturbations of nine order.

From (9), we get

$$r_{1,2} = r_{\pm}^2(\theta, h) = \frac{1 \pm \sqrt{1 - hp(\theta)}}{p(\theta)} \tag{11}$$

and from $\theta' = -1 + r^2 p(\theta) = 0$ we have:

$$\theta_1(h) = \frac{1}{2} \arccos \left[\left(b - a + 2\sqrt{a^2b^2 - ab + (a+b+2ab)\, h^{-1}} \right) / \left(a+b+2ab \right) \right],$$

$$\theta_2(h) = \frac{1}{2} \arccos \left[\left(b - a - 2\sqrt{a^2b^2 - ab + (a+b+2ab)\, h^{-1}} \right) / \left(a+b+2ab \right) \right].$$

From the form of the perturbation terms

$$p(x, y) = x \left(ux^n + vy^n - b\frac{\beta+1}{\mu+1} x^\mu y^\beta - ux^2 \right),$$

$$q(x, y) = y \left(ux^n + vy^n + bx^\mu y^\beta - vy^2 \right)$$

we have

$$\frac{\partial^2 p(x, y)}{\partial x \partial \varepsilon} + \frac{\partial^2 q(x, y)}{\partial y \partial \varepsilon} = (2+n)(ux^n + vy^n) - 3(ux^2 + vy^2) - 2\lambda.$$

Therefore, the four detection functions corresponding to the four closed curves Γ_j^h, $j = 1 - 4$, for the above perturbations are:

$$\lambda_j(h) = \frac{\iint\limits_{\Gamma_j^h(D)} \left[(n+2)(ux^n + vy^n) - 3(ux^2 + vy^2) \right] dxdy}{2 \iint\limits_{\Gamma_j^h(D)} dxdy}, j = 1 - 4 \tag{12}$$

In polar coordinates and for $a = \frac{1}{8}$, $b = \frac{1}{4}$ and $n = 8$, (12) leads to:

$$\lambda_1(h) = \frac{\int\limits_0^{2\pi} \left(r_1^5(\theta,h)\, g(\theta) - \frac{3}{4} r_1^2(\theta,h)\, g_1(\theta) \right) d\theta}{\int\limits_0^{2\pi} r_1(\theta,h)\, d\theta}, \quad -\infty < h < 4, \tag{13}$$

$$\lambda_2(h) = \frac{\int\limits_0^{2\pi} \left(r_2^5(\theta,h)\, g(\theta) - \frac{3}{4} r_2^2(\theta,h)\, g_1(\theta) \right) d\theta}{\int\limits_0^{2\pi} r_2(\theta,h)\, d\theta}, \quad 0 < h < 4, \tag{14}$$

$$\lambda_3(h) = \frac{\int\limits_{-\theta_2(h)}^{\theta_2(h)} \left[\left(r_1^5(\theta,h) - r_2^5(\theta,h) \right) g(\theta) - \frac{3}{4} \left(r_1^2(\theta,h) - r_2^2(\theta,h) \right) g_1(\theta) \right] d\theta}{\int\limits_{-\theta_2(h)}^{\theta_2(h)} \left(r_1(\theta,h) - r_2(\theta,h) \right) d\theta},$$

$$4 < h < 8, \tag{15}$$

$$\lambda_4(h) = \frac{\int\limits_{\theta_1(h)}^{\theta_2(h)} \left[\left(r_1^5(\theta,h) - r_2^5(\theta,h) \right) g(\theta) - \frac{3}{4} \left(r_1^2(\theta,h) - r_2^2(\theta,h) \right) g_1(\theta) \right] d\theta}{\int\limits_{\theta_1(h)}^{\theta_2(h)} \left(r_1(\theta,h) - r_2(\theta,h) \right) d\theta},$$

$$8 < h < 14.4516, \tag{16}$$

where $g(\theta) = u\cos^8\theta + v\sin^8\theta$, $g_1(\theta) = u\cos^2\theta + v\sin^2\theta$ and $r_{1,2}(\theta,h) = r_{\pm}^2(\theta,h)$.

From (13)-(16) we obtain values of detection functions $\lambda_i(h), i = 1-4$ registered in the following tables (1-4).

Table 1

Values of detection function $\lambda_1(h)$ when $a = 1/8, b = 1/4, n = 8$.

h	$\lambda_1(h)$	h	$\lambda_1(h)$	h	$\lambda_1(h)$
−2	81404.61 u+11223.46 v	−1.85	80530.61 u+11076.35 y	−1.7	79659.73 u+10929.97 v
−1.55	78792.00 u+10784.34 v	−1.4	77927.44 u+10639.45 v	−1.25	77066.07 u+10495.30 v
−1.1	76207.91 u+10351.91 v	−0.95	75352.99 u+10209.27 v	−0.8	74501.33 u+10067.40 v
−0.65	73652.95 u+9926.28 v	−0.5	72807.90 u+9785.94 v	−0.35	71966.20 u+9646.38 v
−0.2	71127.89 u+9507.59 v	−0.05	70293.00 u+9369.59 v	0.1	69461.58 u+9232.39 v
0.25	68633.66 u+9095.98 v	0.4	67809.31 u+8960.38 v	0.55	66988.57 u+8825.60 v
0.7	66171.50 u+8691.64 v	0.85	65358.17 u+8558.51 v	1	64548.64 u+8426.23 v
1.15	63743.01 u+8294.80 v	1.3	62941.37 u+8164.24 v	1.45	62143.81 u+8034.55 v
1.6	61350.45 u+7905.76 v	1.75	60561.43 u+7777.88 v	1.9	59776.91 u+7650.93 v
2.05	58997.07 u+7524.93 v	2.2	58222.11 u+7399.90 v	2.35	57452.30 u+7275.88 v
2.5	56687.94 u+7152.90 v	2.65	55929.41 u+7030.99 v	2.8	55177.17 u+6910.22 v
2.95	54431.84 u+6790.64 v	3.1	53694.20 u+6672.34 v	3.25	52965.33 u+6555.42 v
3.4	52246.80 u+6440.04 v	3.55	51541.02 u+6326.44 v	3.7	50852.18 u+6215, 03 v
3.85	50189.31 u+6106, 69 v	4.	49596.01 u+6005, 58 v		

Table 2

Values of detection function $\lambda_2(h)$ when $a = 1/8, b = 1/4, n = 8$.

h	$\lambda_2(h)$	h	$\lambda_2(h)$	h	$\lambda_2(h)$
0.01	−0.001875u−0.001875v	0.21	−0.039486 u−0.03975 v	0.41	−0.076928 u−0.07795 v
0.61	−0.11309 u−0.11530 v	0.81	−0.14607 u−0.14955 v	1.01	−0.17306 u−0.17714 v
1.21	−0.19029 u−0.19278 v	1.41	−0.19289 u−0.18897 v	1.61	−0.17480 u−0.15530 v
1.81	−0.12865 u−0.07747 v	2.01	−0.04560 u+0.06406 v	2.21	0.084763 u+0.29618 v
2.41	0.27464 u+0.65610 v	2.61	0.53813 u+1.19632 v	2.81	0.89138 u+1.99267 v
3.01	1.3526 u+3.158 v	3.21	1.942 u+4.872 v	3.41	2.683 u+7.439 v
3.61	3.597 u+11.44 v	3.81	4.705 u+18.296 v		

Table 3

Values of detection function $\lambda_3(h)$ when $a = 1/8, b = 1/4, n = 8$.

h	$\lambda_3(h)$	h	$\lambda_3(h)$	h	$\lambda_3(h)$
4	59182.00 u+7159.73 v	4.07	59275.89 u+7152.54 v	4.14	59278.05 u+7136.37 v
4.21	59246.41 u+7116.98 v	4.28	59193.17 u+7095.584 v	4.35	59124.06 u+7072.74 v
4.42	59042.48 u+7048.79 v	4.49	58950.65 u+7023.96 v	4.56	58850.16 u+6998.41 v
4.63	58742.21 u+6972.26 v	4.7	58627.74 u+6945.61 v	4.77	58507.48 u+6918.52 v
4.84	58382.07 u+6891.07 v	4.91	58252.01 u+6863.30 v	4.98	58117.76 u+6835.27 v
5.05	57979, 70 u+6807.01 v	5.12	57838.19 u+6778.57 v	5.19	57693.51 u+6749.96 v
5.26	57545.97 u+6721.23 v	5.33	57395.79 u+6692.40 v	5.4	57243.23 u+6663.51 v
5.47	57088.50 u+6634.56 v	5.54	56931.80 u+6605.59 v	5.61	56773.32 u+6576.62 v
5.68	56613.26 u+6547.68 v	5.75	56451.79 u+6518.78 v	5.82	56289.09 u+6489.94 v
5.89	56125.32 u+6461.19 v	5.96	55960.65 u+6432.56 v	6.03	55795.26 u+6404.05 v
6.1	55629.32 u+6375.70 v	6.17	55462.99 u+6347.53 v	6.24	55296.44 u+6319.57 v
6.31	55129.86 u+6291.83 v	6.38	54963.44 u+6264.35 v	6.45	54797.36 u+6237.15 v
6.52	54631.83 u+6210.27 v	6.59	54467.07 u+6183.73 v	6.66	54303.30 u+6157.58 v
6.73	54140.78 u+6131.85 v	6.8	53979.77 u+6106.59 v	6.87	53820.56 u+6081.83 v
6.94	53663.48 u+6057.64 v	7.01	53508.89 u+6034.07 v	7.08	53357.19 u+6011.19 v
7.15	53208.85 u+5989.08 v	7.22	53064.38 u+5967.81 v	7.29	52924.41 u+5947.50 v
7.36	52789.66 u+5928.28 v	7.43	52660.99 u+5910.28 v	7.5	52539.50 u+5893.70 v
7.57	52426.53 u+5878.77 v	7.64	52323.87 u+5865.8 v	7.71	52233.94 u+5855.22 v
7.78	52160.31 u+5847.67 v	7.85	52108.78 u+5844.19 v	7.92	52090.69 u+5846.84 v
7.99	52144.16 u+5862.58 v				

Table 4

Values of detection function $\lambda_4(h)$ when $a = 1/8, b = 1/4, n = 8$.

h	$\lambda_4(h)$	h	$\lambda_4(h)$	h	$\lambda_4(h)$
8	52174.16 u+5868.95 v	8.15	52274.77 u+5899.11 v	8.3	52204.20 u+5896.95 v
8.45	52073.14 u+5883.21 v	8.6	51903.62 u+5862.08 v	8.75	51705.74 u+5835.51 v
8.9	51485.31 u+5804.63 v	9.05	51246.07 u+5770.16 v	9.2	50990.64 u+5732.61 v
9.35	50720.97 u+5692.37 v	9.5	50438.54 u+5649.73 v	9.65	50144.53 u+5604.92 v
9.8	49839.90 u+5558.14 v	9.95	49525.45 u+5509.53 v	10.1	49201.86 u+5459.24 v
10.25	48869.69 u+5407.37 v	10.4	48529.46 u+5354.03 v	10.55	48181.59 u+5299.29 v
10.7	47826.47 u+5243.25 v	10.85	47464.46 u+5185.96 v	11.	47095.85 u+5127.49 v
11.15	46720.94 u+5067.89 v	11.3	46339.97 u+5007.21 v	11.45	45953.18 u+4945.49 v
11.6	45560.78 u+4882.79 v	11.75	45162.97 u+4819.13 v	11.9	44759.93 u+4754.55 v
12.05	44351.83 u+4689.08 v	12.2	43938.83 u+4622.76 v	12.35	43521.08 u+4555.62 v
12.5	43098.71 u+4487.67 v	12.65	42671.85 u+4418.95 v	12.8	42240.63 u+4349.48 v
12.95	41805.16 u+4279.28 v	13.1	41365.56 u+4208.37 v	13.25	40921.91 u+4136.77 v
13.4	40474.33 u+4064.50 v	13.55	40022.91 u+3991.58 v	13.7	39567.72 u+3918.02 v
13.85	39108.87 u+3843.85 v	14.	38646.43 u+3769.06 v	14.15	38180.48 u+3693.69 v
14.3	37710.98 u+3617.74 v	14.45	37231.73 u+3540.61 v		

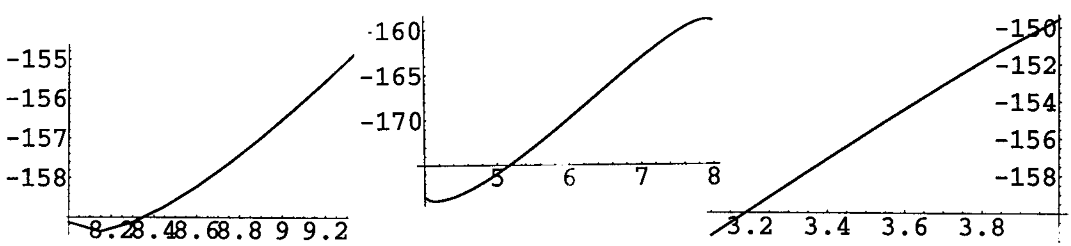

Fig. 4. Detection curves λ_4 (left), λ_3 (middle), λ_1 (right) of system (5) for $a = \frac{1}{8}$, $b = \frac{1}{4}$, $n = 8$, $u = -0.0035$ and $v = 0.004$.

From tables (1-4) we have the four detection functions, three of which are shown in fig. 4. The values of λ_2 are not plotted because they are too small in comparison with the other values. By using Proposition 1.1 and from fig. 4 we have our main result:

Theorem 3.1. *For* $a = \frac{1}{8}, b = \frac{1}{4}, n = 8, u = -0.0035, v = 0.004$ *and* $0 < \varepsilon \ll 1$, *we have that local minimum of* λ_4 *is* -159.365, *first point of* λ_4 *is* $(8, -159.134)$, *last point of* λ_4 *is* $(14.45, -116.149)$, *local minimum of* λ_3 *is* -178.928, *local maximum of* λ_3 *is* -158.93, *first point of* λ_3 *is* $(4, -178.498)$, *last point of* λ_3 *is* $(8, -159.134)$.

Based on these values we have:

a) *If* $-178.928 < \lambda < -178.498$, *system* (5) *has at least five limit cycles, two of which in the neighborhood of each orbit of type* Γ_3^h *and one in the neighborhood of orbit of type* Γ_1^h, *fig.* 5 a)

b) *If* $-178.498 < \lambda < -159.365$, *system* (5) *has at least three limit cycles, one of which in the neighborhood of each orbit of type* Γ_3^h *and one in the neighborhood of orbit of type* Γ_1^h, *fig.* 5 b)

c) *If* $-159.365 < \lambda < -159.134$, *system* (5) *has at least eleven limit cycles, two of which in the neighborhood of each orbit of type* Γ_4^h *and one in the neighborhood of each orbit of type* Γ_3^h *and* Γ_1^h, *fig.* 5 c)

d) $-159.134 < \lambda < -158.93$, *system* (5) *has at least nine limit cycles, one of which in the neighborhood of each orbit of type* Γ_4^h, Γ_1^h *and two of which in the neighborhood of orbit of type* Γ_3^h, *fig.* 6 a)

e) $-158.93 < \lambda < -116.149$, *system* (5) *has at least five limit cycles, one of which in the neighborhood of each orbit of type* Γ_4^h *and* Γ_1^h, *fig.* 6 b).

From fig. 4 we could obtain more results but we listed only these more important cases.

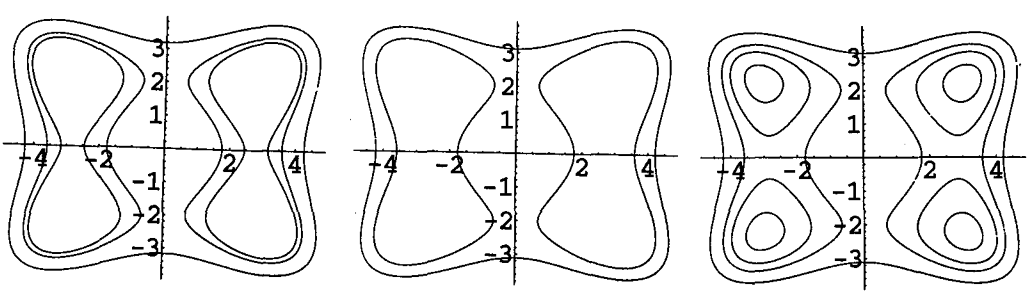

Fig. 5. Distribution diagram corresponding to: a) five (left), b) three (middle), c) eleven (right) limit cycles of system (5)

4. Conclusion

System (5) with $a = \frac{1}{8}$, $b = \frac{1}{4}$, $n = 8$, $u = -0.0035$, $v = 0.004$, $0 < \varepsilon \ll 1$ and $-159.365 < \lambda < -159.134$, has at least eleven limit cycles. The Abelian integral method was employed. By numerically explorations we have drawn the shape of the graphs of the detection functions from which we concluded about the number and distribution of limit cycles. In a forthcoming paper we intend to study the system for $n = 10$ and $n = 12$.

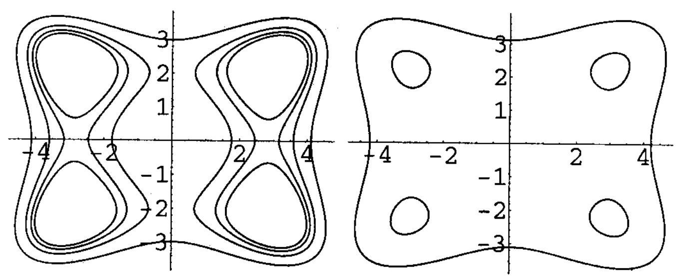

Fig. 6. Distribution diagram corresponding to: a) nine (left) b) five (right) limit cycles of system (5)

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АНАЛИЗИ НА ПЕРТУРБИРАНИ СИСТЕМИ НА ХАМИЛТОН

Gheorghe Tigan

Р е з и м е

Во оваа работа го проучуваме постоењето, бројот и распределбата на пертурбираните Хамилтонови системи дадени со
$x' = 4y\left(abx^2 - by^2 + 1\right) + \varepsilon x(ux^n + vy^n - b\frac{\beta+1}{\mu+1}x^\mu y^\beta - ux^2 - \lambda)$
$y' = 4x\left(ax^2 - aby^2 - 1\right) + \varepsilon y(ux^n + vy^n + bx^\mu y^\beta - vy^2 - \lambda)$
каде $\mu+\beta = n, 0 < a < b < 1, 0 < \varepsilon \ll 1$, u, v, λ се реалните параметри и $n = 2k, k$ е цел број.

Метод на Абелови интеграли е применет [15]. За некои партикуларни параметри ја изведуваме распределбата на ограничени циклични дијаграми.

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