

LINEARIZATION OF A NONLINEAR DIFFERENTIAL EQUATION OF n-TH ORDER

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Abstract. A new proof of Alidema's result [1] is given in this note.

In this note we shall prove the following

Theorem. The equation

$$\sum_{i=1}^n p_i y^{(i)} + \sum_{k=2}^{n-1} \sum_{i=k+1}^n p_i A_{ki} P_k(y) + \sum_{i=2}^n p_i y^{-i} P_i(y) + p_0 P_0(y) = h P_{-1}(y), \quad (1)$$

where

$$A_{ki} = \phi(y', \dots, y^{(i-1)})$$

$$A_{i,i} = y^{(i)}, \quad A_{k,i} = A'_{k,i-1} + y' A_{k-1,i-1}, \quad A_{i,i} = y^{-i} \quad (2)$$

$$P'_{-1} + P_{-1} P_2 = 0, \quad P'_0 + P_0 P_2 = 1, \quad P_i = P'_{i-1} + P_{i-1} P_2, \quad (i=3,4,\dots), \quad (3)$$

has a solution

$$y = f(u) \quad (4)$$

where $u=u(x)$ is a solution of the equation

$$\sum_{i=0}^n p_i u^{(i)} = h \quad (p_i = p_i(x), h = h(x)) \quad (5)$$

and $f(u)$ is any solution of the equation

$$f'(u) = P_{-1}(f(u)). \quad (6)$$

Proof. From (4) we obtain

$$y' = \frac{df}{du} u' \quad (7)$$

or

$$\frac{df}{du} = b(y), \quad u = \int \frac{dy}{b(y)}. \quad (8)$$

Let us take

$$a(t) = \int \frac{dt}{b(t)}, \quad (9)$$

then we have

$$\begin{aligned} u &= a(y), \quad u' = y' a'(y), \quad u'' = y'' a'(y) + y'^2 a''(y), \\ u''' &= y''' a'(y) + 3y' y'' a''(y) + y'^3 a'''(y) \end{aligned} \quad (10)$$

and in the general case [2, p. 250]

$$u^{(i)} = A_{i1} a^{(i)}(y) + \sum_{k=2}^{i-1} A_{ki} a^{(k)}(y) + A_{ii} a^{(i)}(y) \quad (11)$$

with (2). Substituting (10), (11) into (5) and multiply the equation by $1/a'(y)$, we obtain equation (1) with

$$P_i(y) = a^{(i)}(y)/a'(y) \quad (i=2,3,\dots), \quad P_0(y) = a(y)/a'(y), \quad P_{-1}(y) = a/a'(y).$$

Hence, it is clear that the functions P_i, P_0, P_{-1} satisfy (3).

From (9) we have $a'(t)b(t)=1$. Using (4) and left hand side of (8) we obtain equation $f'(u)=b(f(u))$ i.e. equation (6).

R E F E R E N C E S

- [1] R.I.Alidema: The equivalence of certain linear and nonlinear differential equations of n-th order, Publ. Inst. Math. 24 (1978), 5-12.
- [2] И.И.Ляшко, А.К.Боячук, Я.Г.Гай, Г.П.Головач: Математический анализ в примерах и задачах, Т.1, Киев, 1975.

LINEARIZACIJA JEDNE NELINEARNE DIFERENCIJALNE JEDNAČINE n-TOG REDA

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Sadržaj

U noti je dat jedan dokaz Alideminog rezultata.

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