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EQUIVALENCE OF LEBESGUE –STIELTJES MEASURES GENERATED WITH DISTRIBUTION FUNCTION

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Abstract

This paper discusses the equivalence of Lebesgue – Stieltjes measures μ_F and μ_H, generated with probability distribution functions F and $H = F \circ \varphi$ of random variables X and $Y = \varphi^{-1} \circ X$. It is proved that measures μ_F and μ_H are equaivalent in the following sense:

$$\mu_H(B) = 0 \Leftrightarrow \mu_F(B) = 0$$

for every μ_H (i.e. μ_F) negliable set B from σ–algebra $\mathcal{B}$ on $\mathbf{R}$, if every strictly increasing and continuous function φ: $\mathbf{R} \to \mathbf{R}$ satisfies condition

$$\mu_F(B) = 0 \Rightarrow \mu_F\big(\varphi(B)\big) = \mu_F\big(\varphi^{-1}(B)\big) = 0$$

for every μ_F negliable set $B \in \mathcal{B}$.

It is shown that conditions which the function φ: $\mathbf{R} \to \mathbf{R}$ must satisfy for equivalence of measures μ_F and μ_H, $H = F \circ \varphi$ are much simplier if the distribution function F, which generates $L-S$ measure μ_F, is only absolutely continuous, or singular, or discrete.

Finally, a singular probability distribution is constructed and a function $\varphi(x) \not\equiv x$ for which the singular measures μ_F and μ_H, $H = F \circ \varphi$, are equivalent.

Let F be the probability distribution function of the random variable X, defined by:

$$F(x) = P(X < x), \quad x \in \mathbf{R}.$$

Function defined in a such way is increasing left–continuous and satisfies conditions:

$$\lim_{x \to \infty} F(x) = 0 \qquad \text{and} \lim_{x \to \infty} F(x) = 1 .$$

If φ: $\mathbf{R} \to \mathbf{R}$ is a strictly continuous function such that there exists composition $F \circ \varphi$ then $H = F \circ \varphi$ is distribution function of the random variable $Y = \varphi^{-1}X$, because:

$$H(y) = P(Y < y) = P\big(\varphi^{-1}(X) < y\big) = P\big(X < \varphi(y)\big) = F\big(\varphi(y)\big)\,, \quad \forall y \in \mathbf{R}\,.$$

Let μ_F and μ_H Lebesgue – Stieltjes measures generated on the σ–algebra of Borel sets on $\mathbf{R}$ by distribution functions F and H respectively, in a way described in [2].

Definiton 1. *For measures μ_F and μ_H defined on the σ–algebra $\mathcal{B}$, it is said that they are equivalent ($\mu_F \sim \mu_H$) if*

$$\mu_F(B) = 0 \Leftrightarrow \mu_H(B) = 0 \tag{1}$$

for every μ_H as well as for every μ_H negliable set $B \in \mathcal{B}$.

If relation (1) is valid only in one direction, for example if

$$\mu_F(B) = 0 \Rightarrow \mu_H(B) = 0\,, \quad \in \mathcal{B}\,, \tag{2}$$

then it is valid that $L-S$ measure μ_H is absolutely continuous with respect to measure μ_F and is written $\mu_H \ll \mu_F$.

The fact that the relation between $L-S$ measures μ_F and μ_H, $H = F \circ \varphi$ really depends on type and properties of continuous and strictly increasing function φ: $\mathbf{R} \to \mathbf{R}$, will be ilustrated on some simple examples.

Example 1. Let F: $\mathbf{R} \to [0, 1]$ be distribution function of continuous random variable with $\mathbf{R}_x = [a, b]$ and φ: $[a, b] \to [a, b]$ continuous, strictly increasing function that has derivative equal $\emptyset$ almost everywhere, so called singular function. Then function F is absolutely continuous with respect to Lebesgue (L) measure on $\mathbf{R}$. On the other side, function $H = F \circ \varphi$ is singular, so it generates singular $L-S$ measure μ_H with respect to L–measure. Thus, $L-S$ measures μ_F, μ_H in accordance with the definition, are not comparable at all.

Example 2. If

$$F(x) = \begin{cases} 0\,, & x \le 0 \\ \frac{1}{2}x\,, & 0 \le x \le 1 \\ \frac{1}{2}\,, & 1 \le x \le 2 \\ 1 - \frac{1}{x}\,, & x > 2 \end{cases}$$

and $\varphi(x) = x^3 - 3x^2 + 3x$, $x \in \mathbf{R}$, then measures μ_F and μ_H, $H = F \circ \varphi$ are equivalent. If $\varphi(x) = x^3$, $x \in \mathbf{R}$ then μ_F is absolutely continuous with respect to $L-S$ measure μ_H, generated with distribution function:

$$H(x)=(F\circ\varphi)(x)=\begin{cases}0\,, & x\le 0\\ \frac{1}{2}x^3\,, & 0\le x\le 1\\ \frac{1}{2}\,, & 1\le x\le\sqrt[3]{2}\\ 1-\frac{1}{x^3}\,, & x>\sqrt[3]{2}\,.\end{cases}$$

Because, every μ_H negliable set $B\in\mathcal{B}$ and μ_F negliable too

$$\mu_H(B)=0\to\mu_F(B)=0\,,\quad \in\mathcal{B}\,.$$

But reverse is not valid, because if $B=<1,2>$ then

$$\mu_H(B)=0\quad\text{and}\quad\mu_H(B)=H(2)-H(1)=1-\frac{1}{8}-\frac{1}{2}=\frac{3}{8}\,.$$

In order to make the statament of the main theorem more concise, we shall slightly change the definition of so called N–function given by Luzin [1].

Definition 2. *We say that function $\varphi\colon\mathbf{R}\to\mathbf{R}$ has N-property with respect to $L-S$ measure μ_F if $\mu_F(\varphi(B))=0$ for every μ_F negliable set $B\in\mathcal{B}$.*

Theorem 1. *Let F be distribution function of random variable X with $\mathbf{R}_x=<a,b>\subseteq\mathbf{R}$ and $\varphi\colon<a,b>\to<a,b>$ strictly increasing and continuous function, and that exist composition $H=F\circ\varphi$. $L-S$ measures μ_F and μ_H generated with functions F, H respectively, are equivalent if and only if function φ and φ^{-1} have N-property with respect to measure μ_F.*

Proof. Since

$$\mu_H(B)=\int_B dH=\int_B d(F\circ\varphi)=\mu_F(\varphi(B)) \tag{3}$$

then from the supposition that φ and φ^{-1} have N–property with respect to measure μ_F, we obtain:

$$\mu_F(B)=0\to\mu_F(\varphi(B))=0\to\mu_H(B)=0\,,$$

$$\mu_H(B)=0\to\mu_F(\varphi(B))=0\to\mu_F(\varphi^{-1}(\varphi(B)))=0\to\mu_F(B)=0\,.$$

That proves the equivalence of $L-S$ measures μ_F and μ_H, $H=F\circ\varphi$. Also, from the supposition on equivalence of measures μ_F and μ_H, equality (3), for every μ_F negliable set $B\in\mathcal{B}$ follow relations:

$$\mu_F(B)=0\to\mu_H(B)=0\to\mu_F(\varphi(B))=0\,,$$

$$\mu_F(B)=0\to\mu_F(\varphi(\varphi^{-1}(B)))=0\to\mu_H(\varphi^{-1}(B))=0\to\mu_F(\varphi^{-1}(B))=0$$

which prove that function φ and φ^{-1}: $< a,\, b > \to < a,\, b >$ have N–property with respect to $L - S$ measure μ_F.

With that theorem is completly proved.

Because, monotone function F: $\mathbf{R} \to \mathbf{R}$ can be up to the additive constant represented as a sum of a step function F_1, an absolutely continuous function F_2 and a singular function F_3, every $L - S$ measure μ_F has unique representation as a sum:

$$\mu_F = \mu_{F_1} + \mu_{F_2} + \mu_{F_3}$$

where μ_{F_1} is a discrete $L - S$ measure generated with discrete function F_1, μ_{F_2} an absolutely continuous measure generated with absolutely continuous function F_2 and μ_{F_3} a singular $L - S$ measure generated with singular function F_3

Theorem 1. gives necessary and sufficient conditions for the equivalence of $L - S$ measures μ_F and $\mu_{F\circ\varphi}$ in general case. But, if measure μ_F consists only of one component, i.e. is only discrete or absolutely continuous or singular on the whole $\mathbf{R}$, then the given conditions for the function φ can be enormaly simplified, as it will be shown in the further consideration.

Corollary 1. *Let μ_G $L - S$ mesure on $\mathbf{R}$ generated with distribution function of continuous random variable X with $\mathbf{R}_x =< a,\, b > \subseteq \mathbf{R}$. Measures μ_F and μ_H, $H = F \circ \varphi$ are equivalent if strictly increasing and continuous function φ: $< a,\, b > \to < a,\, b >$ satisfies conditions*:

(a) $\quad 0 < \varphi'(x) < \infty$ *almost all on $\mathbf{R} \backslash C$ with respect to Lebesque μ on $\mathbf{R}$*

$$(b) \qquad \varphi(B) \subseteq C \Leftrightarrow B \subseteq C$$

where C is union of the intervals $< \alpha_i,\, \beta_i >$ on which distribution function is constant.

Proof. Distribution function of continuous random variable is absolutely continuous on $\mathbf{R}$, so that corresponding $L - S$ measure μ_F is also absolutely continuous with respect to Lebesgue measure on $\mathbf{R}$. Continuous function with satisfies condition (a) is absolutely continuous and its inverse is an absolutely continuous function. Thus, $F \circ \varphi = H$ is absolutely continuous function wich generates also absolutely continuous measure μ_H with respect to L measure.

To prove the equivalince of these two measures, according to Theorem 1., we must show that functions $\varphi$and φ^{-1} have N–property with respect to μ_F measure. For every μ_F negliable set $B \subseteq \mathbf{R} \backslash C$ the following holds:

$$\begin{cases} \mu_F(B) = 0 \to \mu(B) = 0 \to \mu\big(\varphi(B)\big) = 0 \to \mu_F\big(\varphi(B)\big) = 0\,, \\ \mu_F(B) = 0 \to \mu(B) = 0 \to \mu\big(\varphi^{-1}(B)\big) = 0 \to \mu_F\big(\varphi^{-1}(B)\big) = 0\,, \end{cases} \tag{4}$$

because absolutely continuous functions φ and φ^{-1} have N–property with respect to L measure μ and measures μ_F and μ are equivalent on $\mathbf{R}\backslash C$.

Because of condition (b) we have:

$$\varphi(B) \subseteq < \alpha_i, \beta_i > \quad \text{and} \quad \varphi^{-1}(B) \subset < \alpha_i, \beta_i >$$

so

$$\mu_F\big(\varphi(B)\big) = 0 \quad \text{and} \quad \mu_F\big(\varphi(B)\big) = 0$$

which together with (4) shows N–property of function φ and φ^{-1} with respect to measure μ_F, and it is, according to Theorem 1., necessary and sufficient for equivalence of measures μ_F amd μ_H, $H = F \circ \varphi$.

Remark 1. When range of the continuous random variable is $\mathbf{R}_x = \mathbf{R}$, its distribution function is strictly increasing and absolutely continuous on $\mathbf{R}$, so $C = \emptyset$. In such cases, measures μ_F and $\mu_H = F \circ \varphi$ are equivalent if continuous function φ almost everywhere on $\mathbf{R}$ has positive derivative.

Remark 2. Assumption of the corrolary would not have been correct if instead of condition (a) we would suppose that strictly increasing function φ was absolutely continuous. Namely, the inverse function of the strictly increasing and absolutely continuous function need not be absolutely continuous, what will be illustrated on example 3., so as a monotone function, it need not even have N–property which is a necessary condition for the equivalence of the $L - S$ measures μ_F and $\mu_{F\circ\varphi}$.

Example 3. Let us take a nowhere dense closed set B on $[0, 1]$ of Lebesque measure $1 - \varepsilon$. Such sets for any $\varepsilon > 0$, by [1]. Nowhere dense set means that for any $a, b \in [0, 1]$, $a < b$, exist α, β, and $a \leq \alpha < \beta \leq b$, such that $< \alpha, \beta >$ has no points in B. B is measurable as a closed set. Let $f = \zeta_A$ be characteristic function of set $A = [0, 1] \setminus B$. Because f is a summable function, a new function can be defined with:

$$\varphi(x) = \int_0^x f(t)\, dt\,, \quad x \in [0, 1]\,. \tag{5}$$

Function φ defined in such a way is absolutely continuous as an integral, and its derivates is almost everywhere equal to f. Thus, $\varphi'(x) = 0$ on set B of positive measure $1 - \varepsilon$. Besides, φ is strictly increasing, because for any two points $a.b \in [0, 1]$, $a < b$ we have

$$\varphi(b) - \varphi(a) \geq \varphi(\beta) - \varphi(\alpha) = \beta - \alpha - 0\,.$$

Thus, function φ given with (5) is strictly increasing absolutely continuous on $[0, 1]$ and has inverse function. But, φ^{-1} is not absolutely continuous because $\varphi'(x) = 0$ almost everywhere on set B of positive measure $1 - \varepsilon$, $\varepsilon > 0$.

Corollary 2. *Let random variable X on $\mathbf{R}_x = < a, b > \subset \mathbf{R}$ is given with singular distribution function $F(x) = P(X < x)$, and let*

$$E\{x \in < a, b > \mid F'(x) = 0\}.$$

$L - S$ measures μ_F and μ_H *generated respectively with functions* F and $H = F \circ \varphi$, *are equivalent if strictly increasing and continuous function*

$$\varphi\colon < a, b > \to < a, b >$$

is such that

(a) *has finite derivative on E, and on set $< a, b > \backslash E$ either has positive derivative or its derivative doesn't exist;*

$$(b) \qquad \varphi(x) \in E \Leftrightarrow x \in E$$

Proof. $L - S$ measure generated with singular function $F\colon < a, b > \to [0, 1]$ is singular with respect to L measure on $< a, b >$ and it means that $\mu_F(E) = 0$ and $\mu[< a, b > \backslash E] = 0$. From conditions (a) and (b) we get:

$$H'(x) = \left[F\big(\varphi(x)\big)\right]' = F'\big(\varphi(x)\big) \cdot \varphi'(x) \quad \forall x \in E,$$

and

$$H'(x) > 0 \quad \text{or} \quad H'(x) \quad \text{doesen't exists on} \quad < a, b > \backslash E.$$

From here it follows that almost everywhere on $< a, b >$ is $H'(x) = 0$, so that corresponding $L - S$ measure μ_H is singular with respect to L measure on $\mathbf{R}$ and $\mu_H(E) = 0$. So, both $L - S$ measures are singular with respect to L measure on $< a, b >$ and are concentrated on the same set of measure 0, which means that they are equivalent in the sense of Definition 1.

Now we shall construct singular function F and show that there exist functions $\varphi(x) \not\equiv x$ such measures μ_F and $\mu_{F \circ \varphi}$ are equivalent.

Example 4. Construction of the continuous, strictly increasing function $F\colon [0, 1] \to [0, 1]$ that has derivative equal 0 almost everywhere on $I = [0, 1]$.

Let us define input sequence of continuous and strictly increasing functions with derivatives positive almost everywhere on I.

Let $F_0(x) = x^2$, $x \in I$.

For $n \in \mathbf{N}$ let us define $F_n\colon I \to I$ in this way:

$$\begin{aligned} F_n\left(\frac{k}{2^{n-1}}\right) &= F_{n-1}\left(\frac{k}{2^{n-1}}\right), \quad k = \overline{0, 2^{n-1}} \\ F_n\left(\frac{2k+1}{2^n}\right) &= \frac{1}{4} F_{n-1}\left(\frac{k}{2^{n-1}}\right) + \frac{3}{4} F_{n-1}\left(\frac{k+1}{2^{n-1}}\right), \quad k = \overline{0, 2^{n-1}}. \end{aligned} \tag{6}$$

The endpoints of intervals $\left\langle \frac{k}{2^{n-1}}, \frac{2k+1}{2^n} \right\rangle$ i.e. $\left\langle \frac{2k+1}{2^n}, \frac{k+1}{2^{n-1}} \right\rangle$ are connected with parabolas that have vertices in points

$$\left(\frac{k}{2^{n-1}}, F_n\left(\frac{k}{2^{n-1}}\right)\right) \quad \text{and in points} \quad \left(\frac{2k+1}{2^n}, F_n\left(\frac{2k+1}{2^n},\right)\right).$$

It is not difficult to show that a function $F_n: I \to I$ defined in such a way can be written like this:

$$F_n(x) = \begin{cases} 3x^2 & 0 \le x < \dfrac{1}{n} \\ \dfrac{1}{4^n} \displaystyle\sum_{j=0}^{k-1} 3^{n-r_j} + 3^{n-r_k}\left(x - \frac{k}{2^n}\right)^2, & \dfrac{k}{2^n} \le x < \dfrac{k+1}{2^n} \end{cases}$$

where $k = \overline{1, 2^{n-1}}$ and r_j is the number of is in the binary representation of number $j = 0, 1, 2, \ldots$ (for example $r_0 = 0$, $r_1 = 1$, $r_{2^m} = 1$ $r_{2^m+1} = 2, \ldots, m \in \mathbb{N}$).

Defined function $F_n: I \to I$ is obviously continuous and strictly increasing and has positive derivative in all points of the interval $< 0, 1 >$ except in the points of set

$$E_0 = \bigcup_{n=1}^{\infty} \left\{\frac{k}{2^n}, \quad k = \overline{0, 2^n}\right\}$$

in wich derivarive doesn' t exist. Because

$$0 \le F_n(x) \le F_{n+1}(x) \le 1, \quad \forall x \in I \quad \text{and} \quad N \in \mathbb{N},$$

it follows that sequence $F_0, F_1, \ldots, F_n, \ldots$ converges to some increasing function $F: I \to I$. We shall prove that F is strictly increasing continuous function and that $F'(x) = 0$ almost everywhere on I.

Let x be any point from interval $< 0, 1 >$. Let us take the sequence of intervals $< \alpha_n, \beta_n >$, $n \in \mathbb{N}$ such that

$$< \alpha_n, \beta_n > \subset < \alpha_{n-1}, \beta_{n-1} > \quad \text{and} \quad x \in < \alpha_n, \beta_n >, \quad \text{for any} \quad n \in \mathbb{N},$$

where $\alpha_n = \frac{k}{2^n}$, $\beta_n = \frac{k+1}{2^n}$, $k \in \{0, 1, 2, \ldots, 2^n - 1\}$.

From equality (6) it follows that

$$F_n(\beta_n) - F_n(\alpha_n) = F_n\left(\frac{k+1}{2^n}\right) - F_n\left(\frac{k}{2^n}\right) = \frac{3^{n-r_k}}{4^n}.$$

Because

$$F_n(\alpha_n) = F_n\left(\frac{k}{2^n}\right) = F\left(\frac{k}{2^n}\right) = F(\alpha_n) \quad \text{and} \quad F_n(\beta_n) = F(\beta_n)$$

follows $F(\beta_n) - F(\alpha_n) = \frac{3^{n-r_k}}{4^n}$.

From here it follows that $F(\beta_n) > F(\alpha_n)$ and

$$\lim_{n\to infty} \left[F(\beta_n) - F(\alpha_n)\right] = \lim_{n\to\infty} \frac{3^{n-r_k}}{4^n} = 0\,,$$

what shows that F is continuous and strictly increasing function. Derivative $F'(x)$, $x \in <\alpha_n, \beta_n>$, if it exists, is equal to limit of

$$\frac{F(\beta_n) - F(\alpha_n)}{1/2^n} = \frac{3^{n-r_k}}{2^n}\,, \quad \text{when} \quad n \to \infty\,.$$

This limit is either infinity or is not defined or is equal 0. According to Lebesgue theorem [4], monotone continuous function has finite derivative almost everywhere, so our limit function $F\colon I \to I$, as a stricltly increasing and continuous function, has derivative 0 almost everywhere on I. Thus, limit function of the given convergent monotone sequence of functions $F_0, F_1, \ldots$ is singular.

Example of continuous strictly increasing function $\varphi\colon I \to I$ which maps set E on E, and which has finite derivative on set $E = \{x \in I \mid F'(x) = 0\}$ and on set $I \setminus E$ has positive derivative, it exists, can be given in this way:

$$\varphi(x) = \begin{cases} \frac{1}{4}\,x\,, & 0 \le x < \frac{1}{4} \\ \frac{3}{4}\,x - \frac{1}{8}\,, & \frac{1}{4} \le x < \frac{1}{2}\,. \\ \frac{3}{2}\,x - \frac{1}{2}\,, & \frac{1}{2} \le x < 1 \end{cases}$$

Given function $\varphi\colon I \to I$ is obviously continuous, strictly increasing and $0 < \varphi'(x) < \infty$ for all $x \in I \setminus \{0, \frac{1}{4}, \frac{1}{2}, 1\}$.

Because $\{0, \frac{1}{4}, \frac{1}{2}, 1\} \subset E_0 \subseteq I \setminus E$, strictly increasing and composition $F \circ \varphi = H$, his derivative equal 0 on set E, and on set $I \setminus E$ either has positive derivative or its derivative doesn't exist. Function $H = F \circ \varphi\colon I \to I$ is then singular and

$$\{x \in I \mid H'(x) = 0\} = \{x \in I \mid F'(x) = 0\} = E\,.$$

From here follows that μ_F, μ_H, $H = F \circ \varphi$, are $L - S$ measures on the same set $I \setminus E$ of the Lebesgue measure 0 and that

$$\mu_F(E) = \mu_H(E) = 0$$

what means that singular $L - S$ measures μ_F and μ_H are equivalent.

Corrolary 3. *Let $F(x)$, $x \in \mathbf{R}$ be a distribution function of discrete random variable with $\mathbf{R}_x = \{x_1, x_2, \ldots\}$. $L-S$ measure μ_F and μ_H, generated respectively with function F and $H = F \circ \varphi$, are equivalent in terms of definition* 1. *if increasing and continuous function $\varphi\colon \mathbf{R} \to \mathbf{R}$ bijectively maps set $\mathbf{R}_x$ on $\mathbf{R}_x$. Equivalent discrete measures μ_F and μ_H, $H = F \circ \varphi$ are equal.*

Proof. Distribution function of discrete random variable X is step function with jumps in points of set $\mathbf{R}_x$. Because increasing and continuous function $\varphi\colon \mathbf{R} \to \mathbf{R}$ bijectively maps set $\mathbf{R}_x \subset \mathbf{R}$ on $\mathbf{R}_x$ then step function $H = F \circ \varphi$ with jumps in points $x_k \in \mathbf{R}_x$, $k = 1, 2, \ldots$ Thus functions F and H generate discrete $L-S$ measures μ_F and μ_H on σ–algebra B and then $\mu_F(B_1) = \mu_H(B_1) = 0$ for every set $B_1 \in \mathcal{B}$ which has not any element from $\mathbf{R}_x$, and $\mu_F(B_2) > 0$, $\mu_H(B_2) > 0$ for any $B_2 \in \mathcal{B}$ which contains at least element $x_k \in \mathbf{R}_x$. So μ_F and μ_H, $H = F \circ \varphi$ are equivalent measures.

Also, step functions F and H have the same jump in the point $x_k \in \mathbf{R}_x$, $k = 1, 2, \ldots$ so that for $B_2 \in \mathcal{B}$ is

$$\begin{aligned}\mu_H(B_2) &= \sum_{x_k \in B_2} \left[H(x_k + 0) - H(x_k)\right] = \\ &= \sum_{x_k \in B_2} \left[(F \circ \varphi)(x_k + 0) - (F \circ \varphi)(x_k)\right] = \\ &= \sum_{x_k \in B_2} \left[F(x_k + 0) - F(x_k)\right] = \mu_F(B_2).\end{aligned}$$

From here it follows equality of the equivalent discrete $L-S$ measures μ_F and μ_H, $H = F \circ \varphi$.

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ЕКВИВАЛЕНТНОСТА НА ЛЕБЕГ–СТИЛТЈЕС–ОВИ МЕРИ ГЕНЕРИРАНИ ОД ФУНКЦИИТЕ НА РАЗДЕЛБАТА

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Резиме

Во трудов се разгледува еквивалентноста на Лебег–Стилтјес–ови мери μ_F и μ_H, генерирани од функции на распределба на веројатностите F и $H = F\circ\varphi$ на случајните променливи X и $Y = \varphi^{-1}\circ X$. Се покажува дека мерите μF и μ_H се еквивалентни во следната смисла

$$\mu_H(B) = 0 \Leftrightarrow \mu_F(B) = 0\,.$$

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