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FREE OBJECTS IN A VARIETY OF COMMUTATIVE VECTOR VALUED SEMIGROUPS

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0. Abstract. In [5] constructions of free objects in some classes of (n,m)-semigroups are given. In this paper we give a description of a free object with a given nonempty basis in a class of commutative (n,m)-semigroups defined by a system of identities of the form

$$[x_1^p] = [y_1^q],$$

where $p,q > m$ and $(x_1,\dots,x_p)=(y_1,\dots,y_q)$.

1. Preliminaries. Notations we use in this paper are the same as in [5]. Namely, if A^m is the m-th Cartesian power of A, $A\neq\emptyset$, then an element $(a_1,\dots,a_m)\in A^m$ will be denoted by $a_1\dots a_m$, or a_1^m; a_i^r will denote the element $(a_i,\dots,a_r)\in A^{r-i}$ if $i \le r$, and the empty sequence if $i > r$; $(x_j)^r$ denotes the sequence $\underset{r}{x_j\dots x_j}$.

Let $Q\neq\emptyset$, and m,k be positive integers. A mapping $[\,]:a_1^{m+k} \to [a_1^{m+k}]$ from Q^{m+k} into Q^m, such that

$$[x_1^i[x_{i+1}^{i+m+k}]x_{i+m+k+1}^{m+2k}] = [[x_1^{m+k}]x_{m+k+1}^{m+2k}],$$

for each $x_\nu\in Q$, $1 \le i \le k$, is said to be an associative $(m+k,m)$-operation, and the pair $(Q;[\,])$ an $(m+k,m)$-semigroup ([2]). By GAL (the general associative law for vector valued semigroups ([2])), for every $s \ge 1$, $[\,]$ induces a unique $(m+sk,m)$-operation $[\,]^s:Q^{m+sk} \to Q^m$. We say that the $(m+sk,m)$-semigroup $(Q:[\,]^s)$ is derived by the $(m+k,m)$-semigroup $(Q;[\,])$. These facts suggest to use the notation $[a_1^{m+sk}]$ instead of $[a_1^{m+sk}]^s$.

If, moreover,

$$[x_1^{m+k}] = [x_{\sigma(1)}\dots x_{\sigma(m+k)}]$$

for every permutation σ on $(1,2,\dots,m+k)$, then we say that $(Q,[\,])$ is a commutative $(m+k,m)$-semigroup. Every derived semigroup of a commutative $(m+k,m)$-semigroup is commutative as well (GACL [4]).

Let $(Q,[\,])$ be a commutative $(m+k,m)$-semigroup, such that the identities

$$[(x_1)^{\alpha_1}\ldots(x_r)^{\alpha_r}] = [(x_1)^{\beta_1}\ldots(x_r)^{\beta_r}], \tag{1.1}$$

are valid for $\alpha_\nu, \beta_\nu > 0$, $\Sigma\alpha_\nu \equiv m(\mathrm{mod}\ k)$, $\Sigma\beta_\nu \equiv m(\mathrm{mod}\ k)$, where $(x_\nu)^{\alpha_\nu}$ denotes the sequence $\underbrace{x_\nu\ldots x_\nu}_{\alpha_\nu}$. Denote by $\mathcal{C}_{k,m}$ the class of all commutative (m+k,m)-semigroups which satisfy (1.1).

Let us denote by $\mathcal{C}$ the class of commutative (m+k,m)-semigroups that satisfy the identity

$$[(x_1)^k x_1^{m+k}] = [x_1^{m+k}] \tag{1.1'}$$

It is easy to prove that

<u>1.1°</u>. The class $\mathcal{C}_{k,m}$ is the class of commutative (m+k,m)-semigroups that satisfy (1.1').

<u>1.2°</u>. Let $(Q,[\,])\in\mathcal{C}_{1,m}$, and $[[\,]]$ be an (m+k,m)-operation on Q defined by:

$$[[a_1^{m+k}]] = [a_1^{m+k}] \tag{1.2}$$

Then $(Q,[[\ ]])\in\mathcal{C}_{k,m}$.

Conversly, let $(Q,[[\ ]])\in\mathcal{C}_{k,m}$, and let $[\,]$ be an (m+1,m)-operation defined by

$$[aa_1^m] = [[(a)^k a_1^m]] \tag{1.3}$$

Then $(Q,[\,])\in\mathcal{C}_{1,m}$, and $[[a_1^{m+k}]] = [a_1^{m+k}]$.

<u>Proof</u>. By the general associative and commutative law (GACL) we obtain that $[[\,]]$ is a well defined associative and commutative operation on Q, and the identity (1.1') holds.

To prove the converse of the statement it suffices to prove only that $[\,]$ does not depend on the choice of a, i.e. that

$$[aa_1^m] = [a_i a_1^{i-1} a_{i+1}^m a], \tag{1.4}$$

for every $i\in(1,2,\ldots,m)$.

Namely,

$$[aa_1^m] = [[(a)^k a_1^m]] = [[(a_i)^k a_1^{i-1} a_{i+1}^m a]] = [a_i a_1^{i-1} a_{i+1}^m a] \quad \square$$

We note that using the GACL for commutative vector valued semigroups, it is clear that to any (m+1,m)-semigroup an (m+k,m)-semigroup can be associated, thus we can concider the (m+1,m)-

semigroup $(Q,[\,])\in\mathcal{C}_{1,m}$, as a commutative $(m+k,m)$-semigroup satisfying (1.1′).

A subset $P\subseteq Q$ of an $(m+k,m)$-semigroup $(Q;[[\,]])$ is a <u>subsemigroup</u> iff $[[b_1^{m+k}]]\in P^m$, for every $b_\nu\in P$.

Let $(Q,[[\,]])$, $(Q',[[\,]]')\in\mathcal{C}_{k,m}$. A mapping $\phi:Q\to Q'$ is a <u>homomorphism</u> iff

$$\phi[[a_1^{m+k}]] = [[\phi(a_1)\ldots\phi(a_{m+k})]]'$$

We note that in [2] it is shown that a nonempty intersection of subsemigroups of an $(m+k,m)$-semigroup is a subsemigroup, that a homomorphic image of an $(m+k,m)$-semigroup is an $(m+k,m)$-semigroup, and thus a notion of subsemigroup generated by a nonempty set A can be introduced in a natural way.

A free object with a nonempty basis B in a class $\mathcal{D}$ of $(m+k,m)$-semigroups is introduced in the usual algebraic way, i.e.

$(Q;[\,])\in\mathcal{D}$ is a free object with a basis B iff the following conditions are satisfied:

(i) B generates $(Q;[\,])$;

(ii) if $(Q';[\,]')\in\mathcal{D}$, and $\lambda:Q\to Q'$ is an arbitrary mapping, then there exists a homomorphism $\phi:Q\to Q'$ which is an extension of λ.

Next we note some connections between an $(m+1,m)$-semigroup $(Q;[\,])\in\mathcal{C}_{1,m}$ and the associated $(m+k,m)$-semigroup $(Q;[[\,]])\in\mathcal{C}_{k,m}$. Namely,

<u>1.3°</u>. Let $(Q;[\,])$, $(Q';[\,]')\in\mathcal{C}_{1,m}$, and $(Q;[[\,]])$, $(Q';[[\,]]')$ be the associated $(m+k,m)$-semigroups belonging to $\mathcal{C}_{k,m}$, respectively. Then

(i) P is a subsemigroup of $(Q;[\,])$ iff P is a subsemigroup of $(Q;[[\,]])$.

(ii) $\phi:Q\to Q'$ is a homomorphism from $(Q;[\,])$ into $(Q';[\,]')$ iff ϕ is a homomorphism from $(Q;[[\,]])$ into $(Q';[[\,]]')$.

(iii) A nonempty subset $A\subseteq Q$ generales $(Q;[\,])$ iff A generates $(Q;[[\,]])$.

(iv) $(Q;[\,])$ is a free object with a basis B iff $(Q;[[\,]])$ is a free object with a basis B. □

2. m-dimensional semilattices. Let Q be a nonempty set, F(Q) the family of all finite nonepmty subsets of Q, m be a positive integer, and $F_m(Q)$ the family of all nonempty subsets of Q with not more then m elements. Denote by π the canonical mapping from Q^m into $F_m(Q)$, i.e. $\pi(a_1^m)=\{a_1,\dots,a_m\}$. (In other words $\pi(a_1^m)$ is the content of a_1^m.)

We say that (Q;f) is an m-dimensional groupoid if f is a mapping from F(Q) into Q^m. If, in addition, the following equation

$$f(\pi f(X)\cup Y) = f(X\cup Y) \tag{2.1}$$

holds for every $X,Y\in F(Q)$, then we say that (Q;f) is an m-dimensional semigroup.

The class of m-dimensional semigroups is (in a corresponding sence) equivalent to a class of (m+k,m)-semigroups.

We are now ready to establish some connections between m-dimensional semigroups and vector valued semigroups. The proofs of these statements are quite clear.

2.1°. Let (Q;f) be an m-dimensional semigroup, and let a mapping $[\,]:Q^{m+k}\to Q^m$ be defined by:

$$[a_1^{m+k}] = f(\{a_1,\dots,a_{m+k}\}) \tag{2.2}$$

Then, (Q;[]) is an (m+k,m)-semigroup, such that for every $r,s\geq 1$, $a_1^{m+rk}\in Q^{m+rk}, b_1^{m+sk}\in Q^{m+sk}$, the following implication holds:

$$\pi(a_1^{m+rk}) = \pi(b_1^{m+sk}) \Rightarrow [a_1^{m+rk}] = [b_1^{m+sk}], \tag{2.3}$$

i.e. $(Q;[\,])\ \mathcal{C}_{k,m}$.

Conversely, let $(Q;[\,])\in\mathcal{C}_{k,m}$, i.e. (Q;[]) satisfies the implications (2.3). Let $X=\{a,a_1,\dots,a_p\}\in F(Q)$, where $a,a_\nu\in Q$, $p\geq 1$, and let q be a positive integer such that p+q=m+sk, for some $s\geq 1$. Then, if we define

$$f(X) = [a^q a_1^p], \tag{2.4}$$

a well defined m-dimensional semigroup (Q;f) is obtained, such that the identity (2.2) holds. (In this case we say that (Q;[]) is an (m+k,m)-semigroup associated to the m-dimensional semigroup (Q;f), and vice versa). □

A nonempty subset $A \subseteq Q$ is a <u>subsemigroup</u> of $(Q;f)$ iff $f(X) \in A^m$, for every $X \in F(A)$. Let $(Q;f)$ and $(Q';f')$ be two m-dimensional semigroups and $\phi : Q \to Q'$ a mapping. ϕ is a <u>homomorphism</u> iff $\phi(f(X)) = f'(\phi(X))$, for every $X \in F(Q)$.

<u>2.2°</u>. (i) A nonempty intersection of m-dimensional subsemigroups is an m-dimensional subsemigroup. □

(ii) Homomorphic image of an m-dimensional semigroup is an m-dimensional semigroup.

This last proposition gives us an opportunity to define a subsemigroup of an m-dimensional semigroup $(Q;f)$ generated by a nonempty subset $A \subseteq Q$ in a natural way.

A free object with a basis $B \neq \emptyset$ in the class of m-dimensional semigroups is introduced in the usual way. Namely, let $\mathcal{C}$ be a class of m-dimensional semigroups. We say that $(Q;f) \in \mathcal{C}$ is a free object with a basis B in the class $\mathcal{C}$ iff the following conditions are satisfied:

(i) B generates $(Q;f)$;

(ii) For any m-dimensional semigroup $(Q';f') \in \mathcal{C}$, and any mapping $\lambda : B \to Q'$, there exists a homomorphism $\phi : Q \to Q'$ that extends λ.

The next proposition will explain the motivation of introducing the notion of m-dimensional semilattices.

<u>2.3°</u>. Let $(Q;[\,]), (Q';[\,]') \in \mathcal{C}_{k,m}$, and $(Q;f)$, $(Q';f')$ be the associated m-dimensional semigroups, respectively. Then

(a) P is a subsemigroup of $(Q;[\,])$ iff P is a subsemigroup of $(Q;f)$;

(b) $\phi : Q \to Q'$ is a homomorphism from $(Q;[\,])$ into $(Q';[\,]')$ iff it is a homomorphism from $(Q;f)$ into $(Q';f')$;

(c) $A \neq \emptyset$ generates $(Q;[\,])$ iff A generates $(Q;f)$;

(d) $(Q;[\,])$ is a free object with a basis A in the class $\mathcal{C}_{k,m}$ iff $(Q;f)$ is a free object with a basis A in the class of all m-dimensional semigroups. □

This proposition, together with proposition 1.3 and 1.4 allow us to deal only with m-dimensional semigroups and giving a construction of a free m-dimensional semigroup with a given basis, we will obtain a free object with the same basis in $\mathcal{C}_{k,m}$ and in $\mathcal{C}_{1,m}$, as well.

3. Construction of a free m-dimensional semigroup. To give a construction of a free m-dimensional semigroup with a basis B, we will recall some definitions and results given in [9]. Namely, let $f:F(Q) \to F_m(Q)$ be a mapping. Then we say that $(Q;f)$ is an m-dimensional object. If, further more,

$$f(f(X)\cup Y) = f(X\cup Y), \tag{3.1}$$

for any $X,Y\in F(Q)$, then we say that $(Q;f)$ is an associative m-dimensional object.

Let $B\neq\emptyset$, $B\cap\mathbb{N}=\emptyset$ and let a sequence of sets $(C_\alpha \mid \alpha \geq 0)$ be defined by:

$$C_o = B,\ C_{p+1} = C_p\cup(\mathbb{N}_m\times F(C_p)), \tag{3.2}$$

where $\mathbb{N}_m=\{1,2,\dots,m\}$, and m is a given positive integer, and put $S_B=\cup\{C_p \mid p \geq 0\}$.

Let $y\in S_B$. If p is the least nonnegative integer such that $y\in C_p$, then we write $\chi(y)=p$ and say that p is the hierarchy of y. The hierarchy, $\chi(Y)$ of a set $Y\in F(S_B)$ is the number $\max\{\chi(y) \mid y\in Y\}$.

We will next define a relation α, in $F(S_B)$.

(a) If $X,Y\in F(S_B)$, then: $X\alpha Y \leftrightarrow X\alpha y$, for each $y\in Y$.

Thus, it remains to define the meaning of $X\alpha y$, for $X\in F(S_B)$, and $y\in S_B$, and we will define this relation by induction on the hierarchy of y. (Here we use the notation u for the set $\{u\}$.)

First:

(b) $\chi(y) = 0 \to (X\alpha y \leftrightarrow y\in X)$.

Assume $u=(i,Y)\in S_B$, $\chi(u) = t \geq 1$, and that we have a procedure to determine wether $X\alpha y$, for every $X\in F(S_B)$, $y\in S_B$ such that $\chi(y) < t$. Then $X\alpha u$, iff at least one of the following conditions is satisfied:

(c_1) $u \in X$,

(c_2) $X\alpha y$, for every $y \in Y$.

By induction on hierarchy, it can be easily seen that α is a well defined relation in $F(S_B)$. (If $X\alpha Y$, we say that "X <u>absorbs</u> Y".)

Now we will define a subset R_B of S_B (we will say that R_B is the set of irreducible elements of S_B) as follows:

1) $B \subseteq R_B$;

2) $u=(i,Y) \in R_B$ iff the following conditions are satisfied:

2.1) $Y \in F(R_B)$,

2.2) there does not exist a $z \in Y$, such that $(Y \setminus z)\alpha z$,

2.3) Y does not contain a subset of the form $\{(1,z),(2,z),\ldots,(m,z)\}$.

An $X \in F(R_B)$ is said to be <u>reducible</u> iff it satisfies the following conditions:

2.2') there exists a $z \in X$ such that $(X \setminus z)\alpha z$, or

2.3') there exists a subset of X of the form $\{(1,z),(2,z),\ldots,(m,z)\}$.

$X \subseteq R_B$ is <u>irreducible</u> iff it is not reducible.

The next step is to define an associative object on R_B. For that purpose we need a definition of <u>norm</u>, $||X||$, $X \in F(R_B)$. It is defined by induction on hierarchy, in the following way:

3.1) $||X|| = 0 \Leftrightarrow X \subseteq B$;

3.2) $||(i,X)|| = 1+||X||$;

3.3) If $X=\{x_1,\ldots,x_n\}$, $|X|=n$, then

$$||X|| = ||x_1||+||x_2||+\ldots+||x_n||$$

Now we will define an associative object $(R_B;g)$ as follows:

(i) If $X \in F(R_B)$ is irreducible, then $g(X)=X$.

Assume now that $X \in F(R_B)$ is reducible and for every $Y \in F(R_B)$, such that $||Y|| < ||X||$, an irreducible set $g(Y) \in F(R_B)$ is well defined and the following relation holds:

$$g(Y) \neq Y \Leftrightarrow ||g(Y)|| < ||Y|| \qquad (3.3)$$

Consider, first, the case when 2.2′) is satisfied, and let

$$X = X_{p_1} \cup \ldots \cup X_{p_k}, \qquad (3.4)$$

where $p_1 < \ldots < p_k$, and $x \in X_{p_\nu} \Leftrightarrow X(x) = p_\nu$.

Let s be the greatest number such that

$$X' = X_{p_1} \dot{\cup} \ldots \cup X_{p_s}$$

does not satisfy 2.2′). Then $1 \le s < k$. Denote by Z the set of all $z \in X \setminus X'$, such that $X' \alpha z$, and let $Y = X \setminus Z$. Then we have $Z \neq \emptyset$, $Z \cap Y = \emptyset$ and $||Y|| < ||X||$. Therefore $g(Y) \in F(R_B)$ is a well defined irreducible set, and now we define g(X) by:

(ii) $g(X) = g(Y)$.

We have $||g(X)|| = ||g(Y)|| \le ||Y|| < ||X||$, i.e. (2.3) holds.

Finally, assume that X does not satisfy 2.2′). Then 2.3′) holds, and therefore X has the form

$$X = X' \cup \{(1,Z_1),\ldots,(m,Z_m),\ldots,(1,Z_k),\ldots,(m,Z_k)\},$$

where $X' = \emptyset$ or X' is irreducible and $\nu \neq \lambda \Rightarrow Z_\nu \neq Z_\lambda$. Now we have $||X' \cup Z_1 \cup \ldots \cup Z_k|| < ||X||$, and thus g(X) can be defined by:

(iii) $g(X) = g(X' \cup Z_1 \cup \ldots \cup Z_k)$.

In this case, we also have

$$||g(X)|| = ||g(X' \cup Z_1 \cup \ldots \cup Z_k)|| \le ||X' \cup Z_1 \cup \ldots \cup Z_k|| < ||X||$$

Therefore $g: F(R_B) \to F(R_B)$ is a well defined mapping, such that (1.5) holds for every $Y \in F(R_B)$.

3.1°. $(R_B; g)$ is an associative m-object. □

Now we are ready to give a construction of a free m-dimensional semigroup with a basis B.

3.2°. Let B be a nonempty set, and let R_B and g be define as above. Define a mapping $f: F(R_B) \to (R_B)^m$ by

$$f(X) = (1,g(X))\ldots(m,g(X)) \qquad (3.5)$$

Then we have:

(i) $(R_B; f)$ is a free m-dimensional semigroup with a basis B.

(ii) The identity automorphism of $(R_B;f)$ is the unique automorphism of $(R_B;f)$, which is an extension of the embedding mapping from B into R_B.

(iii) If $(Q;f')$ is a free m-dimensional semigroup with a basis B, then there is a unique isomorphism $\phi:(R_B;f) \to (Q;f')$ which is an extension of the embedding from B into Q.

Proof. It is clear that $(R_B;f)$ is an m-dimensional groupoid. Let $X,Y \in F(R_B)$. Then

$$f(\pi f(X) \cup Y) = f(\{(1,g(X)),\dots,(m,g(X))\} \cup Y) =$$
$$= ((i,g(\{(1,g(X)),\dots,(m,g(X))\} \cup Y)))_{i \in N_m} =$$
$$= ((i,g((g(X)) \cup Y)))_{i \in N_m} =$$
$$= \{(1,g(X \cup Y)),\dots,(m,g(X \cup Y))\} =$$
$$= f(X \cup Y)$$

Thus, (R_B,f) is an m-dimensional semigroup.

Let (S,f) be a subsemigroup of (R_B,f), generated by B. We shall prove, by induction on the hierarchy, that $S=R_B$.

Let $x \in R_B$. If $\chi(x)=0$, then $x \in B \subseteq S$. Suppose that for all $y \in R_B$, such that $\chi(y) < r$, $y \in S$, and let $x=(i,Y)$ is such that $\chi(x)=r$. Then $\chi(Y)=r-1$; thus $Y \subseteq S$. But then $f(Y)=\{(1,Y),\dots,(m,Y)\} \in S^m$ implies $x \in S$.

Let, now, (Q,f') be an m-dimensional semigroup, and $\lambda: B \to Q$ a mapping. By induction on the hierarchy, we define a sequence of mapping ξ_ν in the following way:

$\xi_o = \lambda$. Let ξ_ν be define for all elements of R_B with hierarchy less then $i+1$, such that ξ_j is extension of ξ_{j-1}. We define ξ_{i+1} in the following way:

if $\chi(x) < i$, then $\xi_{i+1}(x) = \xi_i(x)$. Let $\chi(x) = i+1$, and $x=(j,\{y_1,\dots,y_p\})$. Then

$$\xi_{i+1}((j,Y)) = f'_j(\{\xi_i(y_1),\dots,\xi_i(y_p)\})$$

Define a mapping $\xi: R_B \to Q$ by

$$\xi(x) = \xi_i(x) \text{ iff } \chi(x) = i.$$

Then ξ is a homomorphic extension of λ.

Thus, (R_B,f) is a free m-dimensional semigroup with a basis B.

It should be noted here that the class of semilattices is a proper subclass of the class of 1-dimensional semigroups. Namely, the class of corresponding semigroups coincides with the variety of commutative semigroups which satisfy the law:

$$x^2y = xy$$

This suggests to define an m-<u>dimensional semilattice</u> as an m-dimensional semigroup which satisfies the following equality:

$$\pi f(a) = a, \tag{3.6}$$

for every $a \in Q$.

<u>3.3°</u>. Let B be a nonempty set and L_B^1 be defined in the following way

$$M_o = B,\ M_{p+1} = M_p \cup (\mathbb{N}_m \times \{X \in F(M_p) \mid |X| > 1\}),\ L_B^1 = \bigcup_{p>o} M_p$$

If we define a mapping $\ell : F(L_B^1) \to (L_B^1)^m$ by:

$$\ell(X) = \begin{cases} X^m, \text{ if } X \in L_B^1 \\ (1,g(X)),\dots,(m,g(X)), \text{ if } X \in F(L_B^1) \setminus L_B^1 \end{cases}$$

then we obtain a free m-dimensional semilattice with a basis B. □

There is a possibility to change the definition of the notion of m-dimensional semilattices replaceing (2.8) by:

$$(\forall X \in F_m(Q))\, \pi f(X) = X, \tag{3.6'}$$

but in this case, if $m \geq 2$, free m-dimensional semilattices, such that $|B| \geq 2$, would not exist, as the following example shows:

<u>Example 3.4</u>. Let m=2, and B=(a,b), and define operation f, and f' from F(B) into B^2 by:

$$f(\{a\}) = aa,\ f(\{b\}) = bb,\ f(\{a,b\}) = ab,$$

$$f'(\{a\}) = aa,\quad f'(\{b\}) = bb,\quad f'(\{a,b\}) = ba.$$

The mapping $a \to a$, $b \to b$ could not be extended into a homomorphism from (B;f) into (B;f'), as $f(\{a,b\}) \neq f'(\{a,b\})$.

The class of m-dimensional semigroups is in fact a subclass of the class of vector valued semigroups ([2,5]). In a similar way, one could define a special kind of m-dimensional semilattices as a special subclass of fully commutative vector valued semigroups ([4]).

R E F E R E N C E S

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СЛОБОДНИ ОБЈЕКТИ ВО ЕДНА МНОГУКРАТНОСТ КОМУТАТИВНИ ВЕКТОРСКО ВРЕДНОСНИ ПОЛУГРУПИ

Б. Јанева

Р е з и м е

Во овој труд разгледуваме една класа комутативни (n,m)-полугрупи во кои важат идентитетите

$$[(x_1)^{\alpha_1}\dots(x_r)^{\alpha_r}] = [(x_1)^{\beta_1}\dots(x_r)^{\beta_r}],$$

за $\alpha_\nu, \beta_\nu > 0$, $\Sigma\alpha_\nu \equiv m(\mathrm{mod}\ k)$, $\Sigma\beta_\nu \equiv m(\mathrm{mod}\ k)$, $k=n-m > 0$ и даваме конструкција на слободен објект со дадена непразна база.

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