Math. Maced.
Vol. 7 (2009)
81– 88

ON q-LAPLACE TRANSFORMS OF A GENERAL CLASS OF q-POLYNOMIALS AND q-HYPERGEOMETRIC FUNCTIONS

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Abstract. This paper invisage the derivation of a theorem concerning the q-Laplace image of a class of q-polynomial family and its applications in terms of the q-Laplace transforms of various q-polynomials. Results involving the q-Laplace transforms of q-polynomials in terms of the basic Kampé-de-Fériet functions are also deduced.

1. Introduction and Definitions

Recently, Yadav and Purohit [15]-[17] have evaluated the q-Laplace images of number of q-polynomials and basic hypergeometric functions of one and more variables with several interesting special cases. This has pawed a way to further investigate the q-Laplace transform of general class of q-polynomial and basic hypergeometric functions.
Hahn [8], defined the q-analogues of the well known classical Laplace transform:

$$\varphi(s) \;=\; \int_0^\infty e^{-st} f(t)\, dt\,, \tag{1.1}$$

by means of the following two q-integrals:

$${}_qL_s\{f(t)\} = \frac{1}{(1-q)} \int_0^{s^{-1}} E_q(qst) f(t)\, d(t;q), \tag{1.2}$$

and

$${}_qL_s\{f(t)\} = \frac{1}{(1-q)} \int_0^\infty e_q(-st) f(t)\, d(t;q); \quad \mathrm{Re}(s) > 0. \tag{1.3}$$

Where the q-exponential series is defined as:

$$e_q(x) = \sum_{n=0}^{\infty} \frac{x^n}{(q\,;q)_n}\,. \tag{1.4}$$

2000 *Mathematics Subject Classification.* Primary:33D60, Secondary: 44A10, 44A20.
Key words and phrases. q-Laplace transforms, general class of q-polynomials, basic hypergeometric functions, and basic Kampé-de Fériet functions.

And

$$E_q(x) = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n(n-1)/2} x^n}{(q\,;q)_n}\,, \tag{1.5}$$

The basic integrals cf. Gasper and Rahman [7], are defined as:

$$\int_0^x f(t)\,d(t\,;q) = x(1-q)\sum_{k=0}^{\infty} q^k f(xq^k), \tag{1.6}$$

$$\int_x^\infty f(t)\,d(t\,;q) = x(1-q)\sum_{k=1}^{\infty} q^{-k} f(xq^{-k}), \tag{1.7}$$

$$\int_0^\infty f(t)\,d(t\,;q) = (1-q)\sum_{k=-\infty}^{\infty} q^k f(q^k). \tag{1.8}$$

By virtue of the results (1.6), the integral equation (1.2) can be expressed as:

$$\varphi(s) \equiv {}_qL_s\{f(t)\} = \frac{(q;q)_\infty}{s}\sum_{j=0}^{\infty} \frac{q^j f(s^{-1}q^j)}{(q;q)_j}. \tag{1.9}$$

Where the function $\varphi(s)$ is called as q-Laplace transform or q-image of the original functions $f(t)$.
For real, or complex α and $0 < |q| < 1$, the q-shifted factorial is defined by:

$$(\alpha\,;q)_n = \begin{cases} 1, & \text{if } n = 0 \\ (1-\alpha)(1-\alpha q)\cdots(1-\alpha q^{n-1})\,, & \text{if } n \in N. \end{cases} \tag{1.10}$$

Also, for $x \neq 0$, we have

$$[x-y]_\nu = x^\nu \prod_{n=o}^{\infty} \left\{ \frac{(1-\frac{y}{x}q^n)}{(1-\frac{y}{x}q^{\nu+n})} \right\} = x^\nu (\tfrac{y}{x};q)_\nu, \tag{1.11}$$

and

$$\Gamma_q(\alpha) = \frac{(q;q)_\infty (1-q)^{1-\alpha}}{(q^\alpha;q)_\infty}, \tag{1.12}$$

where $\alpha \neq 0, -1, -2, \cdots$.
The q-binomial series is given by

$${}_1\Phi_0\,(\alpha; -\,; q, x) = \frac{(\alpha x; q)_\infty}{(x;q)_\infty}. \tag{1.13}$$

The q-binomial coefficients are given by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{(q;q)_n}{(q;q)_{n-k}(q;q)_k}. \tag{1.14}$$

The generalized basic hypergeometric function cf. Gasper and Rahman [7], is given by

$$ {}_r\Phi_s\left[\begin{array}{l} a_1,\cdots,a_r; \\ b_1,\cdots,b_s; \end{array} q,x\right]=\sum_{n=0}^{\infty}\frac{(a_1,\cdots,a_r\,;q)_n x^n}{(q,b_1,\cdots,b_s\,;q)_n}\left\{(-1)^n q^{\frac{n(n-1)}{2}}\right\}^{(1+s-r)}, \tag{1.15} $$

where, for convergence, we have $0<|q|<1$, $|x|<1$ if $r=s+1$, and for any x if $r\leq s$.

Another form of the generalized basic hypergeometric series ${}_r\Phi_s(.)$ cf. Slater [14], is defined as

$$ {}_r\Phi_s\left[\begin{array}{l} a_1,\cdots,a_r; \\ b_1,\cdots,b_s; \end{array} q,x\right]=\sum_{n=0}^{\infty}\frac{(a_1,\cdots,a_r\,;q)_n x^n}{(b_1,\cdots,b_s\,;q)_n(q;q)_n}. \tag{1.16} $$

The basic analogue of Kampé-de Fériet function cf. Srivastava and Karlsson [13], is defined as

$$ \Phi^{A:\,B;\,B'}_{C:\,D;D'}\left(\begin{array}{l} (a):(b);(b'); \\ (c):(d);(d'); \end{array} q;x,y\right) $$

$$ =\sum_{r,s\geq 0}\frac{\prod\limits_{j=1}^{A}(a_j;q)_{r+s}\prod\limits_{j=1}^{B'}(b_j;q)_r\prod\limits_{j=1}^{B''}(b'_j;q)_s\quad x^r y^s}{\prod\limits_{j=1}^{C}(c_j;q)_{r+s}\prod\limits_{j=1}^{D'}(d_j;q)_r\prod\limits_{j=1}^{D''}(d'_j;q)_s\;\;(q;q)_r(q;q)_s}, \tag{1.17} $$

where, for convergence $|x|<1$, $|y|<1$ and $0<|q|<1$.

The basic sine and cosine series are defined as

$$ \sin_q(x)=\frac{e_q(ix)-e_q(-ix)}{2i}, \tag{1.18} $$

$$ \cos_q(x)=\frac{e_q(ix)+e_q(-ix)}{2}. \tag{1.19} $$

The basic sine hyperbolic and basic cosine hyperbolic series are defined as

$$ \sinh_q(x)=\frac{e_q(x)-e_q(-x)}{2}, \tag{1.20} $$

$$ \cosh_q(x)=\frac{e_q(x)+e_q(-x)}{2}. \tag{1.21} $$

The general classes of family of basic hypergeometric polynomials $f_{n,N}(x;q)$ cf. Srivastava and Agarwal [12], in terms of a sequence $\{S_{j,q}\}_{j=0}^{N}$ of parameters is defined as

$$ f_{n,N}(x;q)=\sum_{j=0}^{[n/N]}\left[\begin{array}{c} n \\ Nj \end{array}\right]S_{j,q}\,x^j, \tag{1.22} $$

where N is positive integer and $n=0,1,2,\cdots$.

On suitable specialization of the sequence of arbitrary parameters $S_{j,q}$, the q-Polynomial family $f_{n,N}(x;q)$ yields a number of known q-Polynomials cf. Gasper and Rahman [7], as its special cases. These include namely, the q-Rogers-Szegö polynomials, the discrete q-Hermite polynomials, the Generalized Stieltjes-Weigert

polynomials, the q-Bessel polynomials of second type, the q-Laguerre polynomials, the q-Jacobi polynomials, the q-Konhauser polynomials and several others. We mention the definitions of some of these polynomials as under
The q-Rogers- Szegö polynomials

$$h_n(x\,;\; q) = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q x^k. \tag{1.23}$$

The discrete q-Hermite polynomials

$$H_n(x\,;\; q) = \sum_{k=0}^{[n/2]} \frac{(q;q)_n(-1)^k q^{k(k-1)} x^{n-2k}}{(q^2;q^2)_k (q;q)_{n-2k}}. \tag{1.24}$$

The Generalized Stieltjes-Weigert polynomials

$$S_n(x,p;q) = (-1)^n q^{-n(2n+1)/2}(p;q)_n \sum_{j=0}^{n} \begin{bmatrix} n \\ j \end{bmatrix}_q \cdot \frac{q^{j^2}(-q^{1/2}x)^j}{(p;q)_j}. \tag{1.25}$$

The q-Bessel polynomials of second type

$$y_n(x;\; \alpha/q^2) = q^{n(n-1)/2}{}_2\Phi_1 \left[\begin{matrix} q^{-n},\; q^{\alpha+n-1}\,; \\ -q\,; \end{matrix} \quad q\,,\, -2xq \right]. \tag{1.26}$$

We shall also use the following definitions of various q-Polynomials and basic hypergeometric functions cf. Gasper and Rahman [7], Jain and Srivastava [9], Koelink and Swarttouw [10], in the sequel.
The Affine q-Krowtchauck polynomials

$$K_n^{Aff}(x;a,N;q) \;=\; {}_3\Phi_2 \left[\begin{matrix} q^{-n},\; x,\; 0; \\ aq\,,\; q^{-N}; \end{matrix} \quad q,\; q \right]. \tag{1.27}$$

The Haln-Exton q-Bessel function $J_v(x\,;\,q)$

$$J_v(x\,;\; q) = \frac{x^v(q^{v+1}\,;q)_\infty}{(q:q)_\infty} \cdot {}_1\Phi_1 \left[\begin{matrix} 0; \\ q^{v+1}; \end{matrix} \quad q\,,\; qx^2 \right]. \tag{1.28}$$

The big q-Jacobi polynomials

$$P_n^{(\alpha,\beta)}(x;\gamma,\delta;q) = \frac{(\alpha q, -\delta\alpha q/\gamma;q)_n(\gamma/\alpha q)^n}{(q,-q;q)_n} \cdot$$

$${}_3\Phi_2 \left[\begin{matrix} q^{-n},\; \alpha\,\beta\, q^{n+1},\; \alpha\, xq/\gamma\,; \\ \alpha q,\; -\delta\alpha\, q/\gamma\;; \end{matrix} \quad q,\; q \right]. \tag{1.29}$$

The q-Lommel polynomials

$$R_{m,v}(x\,;\,q) = \sum_{n=0}^{m} \frac{x^{2n-m}(q^{n+1};q)_\infty (q^v;q)_\infty}{(q;q)_\infty (q^{v+m-n};q)_\infty} \cdot {}_2\Phi_1 \left[\begin{matrix} q^{-n},\; q^{v+m-n}\,; \\ q^v\;; \end{matrix} \quad q\,,\, q^{n+1} \right]. \tag{1.30}$$

The q-Bessel function of second type is given by

$$J_{-v}(x;q) = \frac{e^{iv\pi}(q^{v+1};q)_\infty x^{-v} q^{v(v-1)/2}}{(q;q)_\infty} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k q^{k(k+1)/2} x^{2k}\, q^{-vk}}{(q^{-v+1},q)_k (q,q)_k}. \tag{1.31}$$

2. Main Result

This section envisage to derive a theorem involving the q-Laplace transforms of the general class of q-Polynomials and certain basic hypergeometric functions. Interestingly, some of the results are obtained in terms of the q-analogue of the Kampé-de-Fériet functions.

Theorem 1. *Let $f_{n,N}(x^k;q)$ be the family of q-Polynomials defined in terms of a sequence $S_{j,q}(.)$ of complex coefficients, then the following result involving the q-Laplace transform of the x^λ-weighted family of q-Polynomials holds:*

$$ {}_qL_s\left\{x^\lambda f_{n,N}(x^k;q)\right\} = \frac{(1-q)^\lambda}{s^{\lambda+1}} \sum_{j=0}^{[n/N]} \left[\begin{array}{c} n \\ Nj \end{array}\right]_q S_{j,q} \left(\frac{1-q}{s}\right)^{kj} \Gamma_q(kj+\lambda+1), \tag{2.1} $$

where $\mathrm{Re}(k+\lambda+1)>0,\ \lambda>0,\ k\in I,$ *and* N *is a positive integer.*

Proof. We employ (1.9) and (1.22) in the left hand side of the result (2.1), to obtain

$$ {}_qL_s\left\{x^\lambda f_{n,N}(x^k;q)\right\} = \frac{(q;q)_\infty}{s^{\lambda+1}} \sum_{i=0}^{\infty} \frac{q^{i(1+\lambda)}}{(q;q)_i} \cdot \sum_{j=0}^{[n/N]} \left[\begin{array}{c} n \\ Nj \end{array}\right]_q S_{n,q} \left(\frac{q^i}{s}\right)^{kj}, $$

On interchanging the order of summations and summing the resulting inner ${}_0\Phi_0(.)$ series with the help of a result Gasper and Rahman [7], namely,

$$ {}_0\Phi_0(-;-;q,x) = \frac{1}{(x;q)_\infty}, \tag{2.2} $$

we obtain

$$ \frac{(q;q)_\infty}{s^{1+\lambda}} \sum_{j=0}^{[n/N]} \frac{S_{n,q}}{(q^{1+kj+\lambda};q)_\infty} \cdot \left[\begin{array}{c} n \\ Nj \end{array}\right]_q \left(\frac{1}{s}\right)^{kj}, $$

This, after certain simplifications reduces to the right hand side of (2.1).

$$ \frac{(1-q)^\lambda}{s^{\lambda+1}} \sum_{j=0}^{[n/N]} \left[\begin{array}{c} n \\ Nj \end{array}\right]_q S_{n,q} \left(\frac{1-q}{s}\right)^{kj} \Gamma_q(kj+\lambda+1). \tag{2.3} $$

□

3. Special Cases

It is interesting to observe that in view of the definitions (1.23)-(1.26), the Theorem 2.1 leads to the q-Laplace transforms of the above mentioned polynomials after implementing the necessary changes in the values of $S_{j,q}$, N and k. We illustrate the following cases:

(i) If we take $N=1$, $k=1$ and $S_{j,q}=(q;q)_0$ in Theorem 2.1, we obtain the q-Laplace transform of the q-Rogers-Szegö polynomial $h_n(x\,;\,q)$ as

$$ {}_qL_s\left\{x^\lambda\, h_n(x\,;\ q)\right\} = \frac{1}{s^{\lambda+1}} \sum_{j=0}^{n} \left[\begin{array}{c} n \\ j \end{array}\right]_q (q;q)_j (q^{1+j};q)_\lambda (1/s)^j. \tag{3.1} $$

(ii) Again if we take $N = 2$, $k = -2$, $\lambda = n + \mu$ and $S_{j,q} = (q; q^2)_j (-1)^j q^{j(j-1)}$ in the theorem (2.1), we obtained the q-Laplace transform of the discrete q-Hermite polynomial $H_n(x\,;\,q)$ as:

$$ {}_qL_s\left\{ x^{\mu+n}\, H_n(x\,;\; q)\right\} = \frac{(q;q)_{n+\mu}}{s^{n+\mu+1}} \sum_{j=0}^{n/2} \frac{(q^{-n};q)_{2j}(-s^2)^j q^{j(j-2\mu-1)}}{(q^2;q^2)_j (q^{-\mu};q)_{2j}}. \tag{3.2}$$

(iii) On setting $N = 1$, $k = 1$ and $S_{j,q} = \dfrac{(-1)^{n+j} q^{\frac{-n(2n+1)}{2}+j^2+\frac{j}{2}} (p;q)_n}{(p;q)_j}$ in the main result (2.1), we obtain the q-Laplace transform of the Generalized Stieltjes-Weigert polynomial $S_n(x;p;q)$ as:

$$ {}_qL_s\left\{ x^{\lambda} S_n(x;p,q)\right\} = \frac{(-1)^n q^{\frac{-n(2n+1)}{2}} (p;q)_n (q;q)_\lambda}{s^{\lambda+1}} \cdot {}_2\Phi_2\left[\begin{array}{c} q^{-n}, q^{1+\lambda}; \\ p\,, \quad 0\;; \end{array} \; q,\; -\frac{q^{\,n+\frac{3}{2}}}{s}\right]. \tag{3.3}$$

(iv) If we take $N = 1$, $k = 1$ and $S_{j,q} = \dfrac{(q^{\alpha+n-1};q)_j (2q)^j q^{\frac{n(n-1)}{2}-nj+\frac{j(j-1)}{2}}}{(-q;q)_j}$ in the Theorem 2.1, we obtain the q-Laplace transform of the q-Bessel function of second type $y_n(x;\alpha/q^2)$ as

$$ {}_qL_s\left\{ x^{\lambda} y_n(x;\alpha/q^2)\right\} = \frac{q^{\frac{n(n-1)}{2}}(q;q)_\lambda}{s^{\lambda+1}} \sum_{j=0}^{n} \frac{(q^{-n};q)_j (q^{\alpha+n-1};q)_j (-2q/s)^j (q^{\lambda+1};q)_j}{(q;q)_j(-q;q)_j}. \tag{3.4}$$

Similarly, one can deduce a number of known results due to Yadav and Purohit [15], involving the q-Laplace images of a variety of q-polynomials as the applications of the Theorem 2.1.

4. q-Laplace Transforms of Basic Hypergeometric Functions and q-Polynomials

In the following table, we enumerate the q-Laplace transforms of certain basic hypergeometric functions and q-polynomials. Some of the results deduced, are expressible in terms of the q-analogue of the Kampé-de Fériet functions.

Eq.No.	$f(t)$	$\varphi(s) \equiv {}_qL_s\{f(t)\} = \frac{1}{(1-q)} \int_0^{s^{-1}} E_q(qst) f(t)\, d(t;q);\, Re(s) > 0$
4.1	$x^{\lambda};\ \lambda > 0$	$\frac{(q;q)_\lambda}{s^{1+\lambda}}$
4.2	$x^{\nu} e_q(ax^k);\ k \in$	$\frac{(q;q)_\nu}{s^{1+\nu}} \sum_{r=0}^{\infty} \frac{\left(a/s^k\right)^r (q^{1+\nu}, q^{1+\nu+1}, \cdots, q^{1+\nu+k-1}; q^k)_r}{(q;q)_r}$
4.3	$e_q(x)\, {}_r\Phi_s\left[\begin{array}{c} a_1, \cdots, a_r; \\ b_1, \cdots, b_s; \end{array} q, tx\right]$	$\frac{1}{s}\, \Phi \begin{array}{c} 1:0;\, r \\ 0:0;\, s \end{array} \left(\begin{array}{c} q: -; a_1, \cdots, a_r; \\ -:-\,; b_1, \cdots, b_s; \end{array} q\,; \frac{1}{s}, \frac{t}{s}\right)$
4.4	$\sin_q(x)\, {}_r\Phi_s\left[\begin{array}{c} a_1, \cdots, a_r; \\ b_1, \cdots, b_s; \end{array} q, tx\right]$	$\frac{1}{2is}\, \Phi \begin{array}{c} 1:0;\, r \\ 0:0;\, s \end{array} \left(\begin{array}{c} q: -; a_1, \cdots, a_r; \\ -:-\,; b_1, \cdots, b_s; \end{array} q\,; \frac{i}{s}, \frac{t}{s}\right)$ $-\frac{1}{2is}\, \Phi \begin{array}{c} 1:0;\, r \\ 0:0;\, s \end{array} \left(\begin{array}{c} q: -; a_1, \cdots, a_r; \\ -:-\,; b_1, \cdots, b_s; \end{array} q\,; \frac{-i}{s}, \frac{t}{s}\right)$
4.5	$\cos_q(x)\, {}_r\Phi_s\left[\begin{array}{c} a_1, \cdots, a_r; \\ b_1, \cdots, b_s; \end{array} q, tx\right]$	$\frac{1}{2s}\, \Phi \begin{array}{c} 1:0;\, r \\ 0:0;\, s \end{array} \left(\begin{array}{c} q: -; a_1, \cdots, a_r; \\ -:-\,; b_1, \cdots, b_s; \end{array} q\,; \frac{i}{s}, \frac{t}{s}\right)$ $\frac{1}{2s}\, \Phi \begin{array}{c} 1:0;\, r \\ 0:0;\, s \end{array} \left(\begin{array}{c} q: -; a_1, \cdots, a_r; \\ -:-\,; b_1, \cdots, b_s; \end{array} q\,; \frac{-i}{s}, \frac{t}{s}\right)$

4.6	$\sinh_q(x)\, {}_r\Phi_s\left[\begin{matrix} a_1,\cdots,a_r; \\ b_1,\cdots,b_s; \end{matrix}\ q, tx\right]$	$\frac{1}{2s}\ \Phi\begin{matrix}1:0;\,r\\0:0;\,s\end{matrix}\left(\begin{matrix} q:-;a_1,\cdots,a_r; \\ -:-;b_1,\cdots,b_s; \end{matrix}\ q;\frac{1}{s},\frac{t}{s}\right)$ $-\frac{1}{2s}\ \Phi\begin{matrix}1:0;\,r\\0:0;\,s\end{matrix}\left(\begin{matrix} q:-;a_1,\cdots,a_r; \\ -:-;b_1,\cdots,b_s; \end{matrix}\ q;\frac{-1}{s},\frac{t}{s}\right)$
4.7	$\cosh_q(x)\, {}_r\Phi_s\left[\begin{matrix} a_1,\cdots,a_r; \\ b_1,\cdots,b_s; \end{matrix}\ q, tx\right]$	$\frac{1}{2s}\Phi\begin{matrix}1:0;\,r\\0:0;\,s\end{matrix}\left(\begin{matrix} q:-;a_1,\cdots,a_r; \\ -:-;b_1,\cdots,b_s; \end{matrix}\ q;\frac{1}{s},\frac{t}{s}\right)$ $+\frac{1}{2s}\ \Phi\begin{matrix}1:0;\,r\\0:0;\,s\end{matrix}\left(\begin{matrix} q:-;a_1,\cdots,a_r; \\ -:-;b_1,\cdots,b_s; \end{matrix}\ q;\frac{-1}{s},\frac{t}{s}\right)$
4.8	$(x+a)_\nu$	$\frac{(q;q)_\nu}{s^{1+\nu}(-as;q)_\infty}$ or $\frac{(q;q)_\nu}{s^{1+\nu}}\,{}_0\Phi_0\left[\begin{matrix} -; \\ -; \end{matrix}\ q,-as\right]$
4.9	$x^\lambda(x+a)_\nu;\ \lambda>0$	$\frac{(q;q)_{\nu+\lambda}}{s^{1+\nu+\lambda}}\,{}_1\Phi_1\left[\begin{matrix} q^{-\nu}; \\ q^{-\nu-\lambda}; \end{matrix}\ q\ ,\ as/q\right]$
4.10	$K_n^{Aff}(x;a,N;q)$ $n>N$	$\frac{(q;q)_\infty}{s}\ \Phi\begin{matrix}1:0;\,2\\0:1;\,2\end{matrix}\left[\begin{matrix} 1/s:-;\ q^{-n},\,0; \\ -:1/s;aq,\ q^{-N}; \end{matrix}\ q,\ q,\ q\right]$
4.11	$J_v(x;\ q)$	$\frac{1}{s^{1+\nu}}\cdot{}_4\Phi_4\left[\begin{matrix} q^{\frac{v+1}{2}},-q^{\frac{v+1}{2}},q^{\frac{v+2}{2}},-q^{\frac{v+2}{2}}; \\ q^{v+1},\ 0,\ 0,\ 0; \end{matrix}\ q\ ,\ \frac{q}{s^2}\right]$
4.12	$J_{-v}(x;q)$	$\frac{e^{iv\pi}(q;q)_{-\nu}}{(q;q)_\nu s^{1-\nu}}\cdot{}_4\Phi_4\left[\begin{matrix} q^{\frac{1-\nu}{2}},-q^{\frac{1-\nu}{2}},q^{\frac{2-\nu}{2}},-q^{\frac{2-\nu}{2}}; \\ q^{1-\nu},\ 0\ ,\ 0\ ,\ 0; \end{matrix}\ q,\ \frac{q^{-\nu+1}}{s^2}\right]$
4.13	$P_n^{(\alpha,\beta)}(x;\ \gamma,\delta;q)$	$\frac{(\alpha q,\ -\delta\alpha\, q/\gamma,q)_n(\gamma/\alpha\, q)^n(q;q)_\infty}{(q,-q;q)_n s^{1+\rho}}$ $\Phi\begin{matrix}1:0;2\\0:1;2\end{matrix}\left[\begin{matrix} \frac{q\alpha}{s\gamma}:-;\ q^{-n},\ \alpha\beta\, q^{n+1}; \\ -:\frac{q\alpha}{s\gamma};\alpha q,\ \frac{-\delta\alpha q}{\gamma}; \end{matrix}\ q,q^{1+\rho},q\right]$
4.14	$R_{m,v}(x\ ;\ q)$	$\frac{(q;q)_{-m}(q^\nu;q)_m}{s^{1-m}}\,{}_2\Phi_1\left[\begin{matrix} q^{-n},q^{\nu+m-n}; \\ q^\nu; \end{matrix}\ q,q^{n+1}\right]$ ${}_4\Phi_4\left[\begin{matrix} q^{\frac{1-m}{2}},-q^{\frac{1-m}{2}},q^{\frac{2-m}{2}},-q^{\frac{2-m}{2}}; \\ q^{1-\nu-m},\ 0\ ,\ 0\ ,\ 0\ ; \end{matrix}\ q,\ \frac{q^{1-\nu-m}}{s^2}\right]$

To prove the result (4.2), we take $f(x)=x^\nu e_q(ax^k)$ in the equation (1.9) and make use the definition (1.4), which yields

$$ {}_qL_s\left\{\,x^\nu e_q(ax^k)\right\}=\frac{(q;q)_\infty}{s^{1+\nu}}\sum_{j=0}^{\infty}\frac{q^{j(1+\nu)}}{(q;q)_j}\sum_{r=0}^{\infty}\frac{\left\{a(s^{-1}q^j)^k\right\}^r}{(q,q)_r}. $$

On interchanging the order of summations and then summing the resulting ${}_0\Phi_0(.)$ series with the help of equation (2.2), the right hand side of the above expression

$$ \frac{(q;q)_\infty}{s^{1+\nu}}\sum_{r=0}^{\infty}\frac{\left(a/s^k\right)^r(q^{1+\nu},q^{1+\nu+1},\cdots,q^{1+\nu+k-1};q^k)_r}{(q;q)_r} \tag{4.15} $$

For the proof of the result (4.3), we take $f(x)=e_q(x)\,{}_r\Phi_s\left[\begin{matrix} a_1,\cdots,a_r; \\ b_1,\cdots,b_s; \end{matrix}\ q,tx\right]$ in the equation (1.9) and make use of definition (1.4) and (1.16), this yields;

$$ {}_qL_s\left\{e_q(x)\,{}_r\Phi_s\left[\begin{matrix} a_1,\cdots,a_r; \\ b_1,\cdots,b_s; \end{matrix}\ q,tx\right]\right\}= $$
$$ \frac{(q;q)_\infty}{s}\sum_{j=0}^{\infty}\frac{q^j}{(q;q)_j}\sum_{r=0}^{\infty}\frac{(s^{-1}q^j)^r}{(q;q)_r}\sum_{k=0}^{\infty}\frac{(a_1,\cdots,a_r;q)_k(ts^{-1}q^j)^k}{(q,b_1,\cdots,b_s;q)_k} $$

On interchanging the order of summations and then summing the inner ${}_0\Phi_0(.)$ series with the help of equation (2.2), the right hand side of the above expression reduces

$$ \frac{(q;q)_\infty}{s}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\frac{(a_1,\cdots,a_r;q)_k(t/s)^k(1/s)^r}{(q^{1+r+k};q)_\infty(q;q)_r(q,b_1,\cdots,b_s;q)_k} \tag{4.16} $$

On further simplification in the above expression we get the result (4.3). Proofs of the results (4.4) - (4.9) follow similarly.
To prove the result (4.10), we take $f(x) = K_n^{Aff}(x; a, N; q)$ in the equation (1.9) and make use of the definition (1.27), which yields This further simplifies to the right hand side of the result (4.10).
Proof of the results (4.11) - (4.14) follows similarly. We avoid the proofs for the sake of brevity.

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